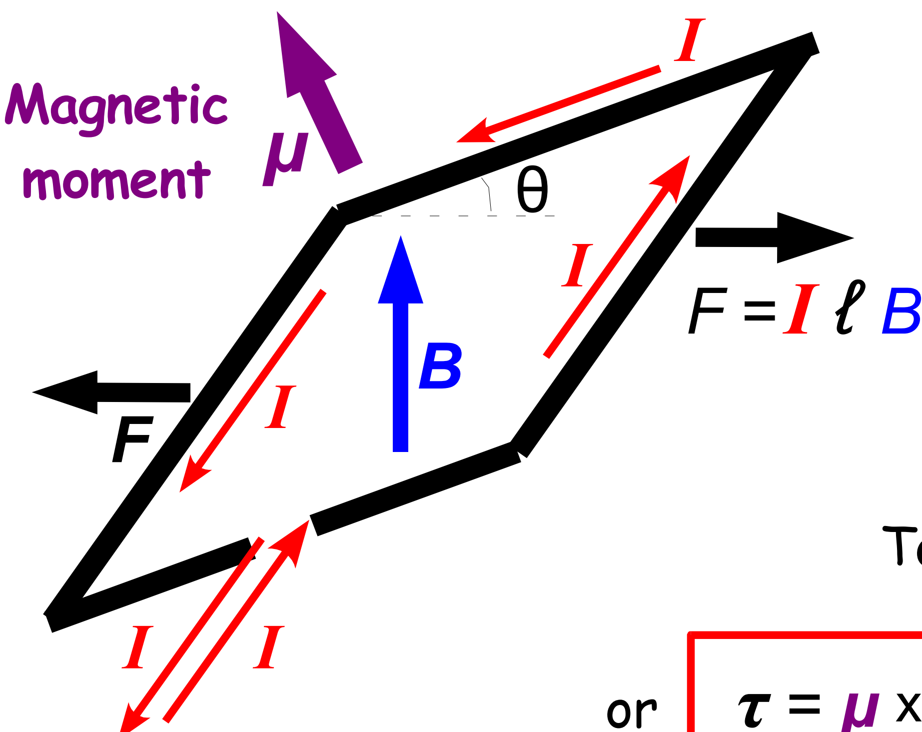
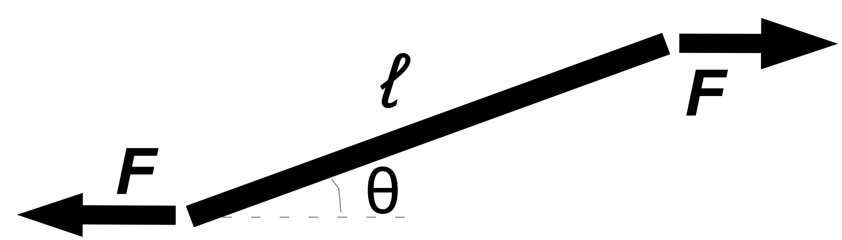
Torque on a Current Loop

$$d\boldsymbol{F} = I\, d\boldsymbol{\ell} \times \boldsymbol{B}$$

Picture a square loop ℓ on a side in a uniform magnetic field $\boldsymbol{B}$:



View down axis of symmetry:



Torque $|\boldsymbol{\tau}| = \ell F \sin\theta = I \ell^2 B \sin\theta$

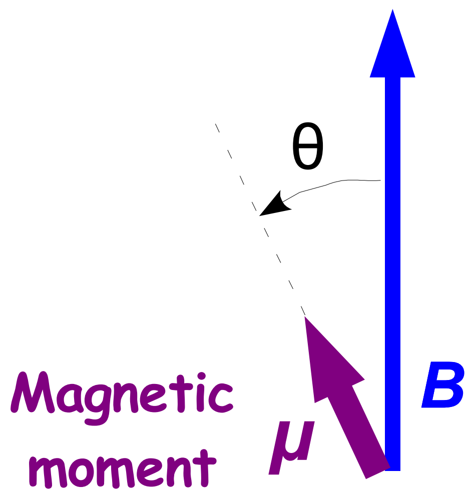
or $\boxed{\boldsymbol{\tau} = \boldsymbol{\mu} \times \boldsymbol{B}}$ where $\boxed{\boldsymbol{\mu} = I \boldsymbol{A}}$ and $\boldsymbol{A} = \ell^2 \hat{\boldsymbol{n}}$

is the loop's **area** times a unit normal vector, **independent** of the loop's **shape**.

Energy of a Magnetic Moment

The torque $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$ "tries" to rotate $\boldsymbol{\mu}$ until it is parallel with $\mathbf{B}$.

As usual we calculate **angular work** as $dW = \boldsymbol{\tau}\, d\theta$. If $d\theta$ is in the direction shown,



dW is negative and the potential energy change $dU = -\,dW$ is positive. Since $|\boldsymbol{\tau}| = \mu B \sin\theta$ is a function of θ, we must integrate $dU = \mu B \sin\theta\, d\theta$ or $dU = -\,\mu B\, du$ where $u \equiv \cos\theta$ to get $U = -\,\mu B \cos\theta$.

This can be written

$$U = -\,\boldsymbol{\mu} \cdot \mathbf{B}$$

This expression should be familiar from Thermal Physics.

(Electrons, being negatively charged, have μ opposite to their spin.)