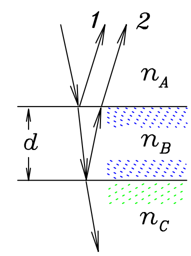
THE UNIVERSITY OF BRITISH COLUMBIA

Physics 108 Assignment # 11 SOLUTIONS: INTERFERENCE

Wed. 23 Mar. 2005 — finish by Wed. 30 Mar.

1. NON-REFLECTIVE FILM COATING: A sheet of glass having an index of refraction of 1.35 is to be coated with a film of material having a refractive index of 1.5 such that bluish-green light (wavelength = 500 nm) is preferentially transmitted.



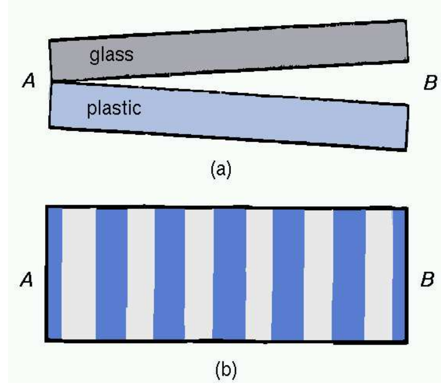
(a) What is the minimum thickness of the film that will achieve the desired result?
ANSWER: Reflected ray 1 picks up a phase shift $\Delta\phi_1 = \pi$ because it reflects off a "denser" (higher n) medium. Ray 2 has no such shift because it reflects off a *less* dense (lower n) medium; however, it goes further than ray 1 by $\Delta\ell = 2d$ (remember, all rays are presumed to have normal incidence, even though they are sketched obliquely for clarity) and therefore it is out of phase with ray 1 by an extra $\Delta\phi_2 = 2\pi\Delta\ell/\lambda_B = \pi(4dn_B/\lambda)$ where $\lambda_B = \lambda/n_B$ is the wavelength in medium A. Therefore the two rays are out of phase by a total of

$$\Delta\phi_{12} = \Delta\phi_2 - \Delta\phi_1 = \pi\left(\frac{4dn_B}{\lambda} - 1\right) = \pi\left(\frac{4 \times d \times 1.5}{500\ nm} - 1\right) \qquad \text{altogether.}$$

For a maximum *transmission* at λ we want *minimum reflection* — *i.e.* *destructive* interference between rays 1 and 2, which occurs when $\Delta\phi_{12}$ is an *odd* multiple of π. In principle, *no thickness at all* ($d \to 0$ or at least $d \ll \lambda$) would give maximum transmission, but that would work *equally* well for *all* wavelengths, whereas the question specifies "…*preferentially* transmitted." Only for a nonzero film thickness can there be any dependence of $\Delta\phi_{12}$ upon λ. (Also, it's hard to make a film of zero thickness!) Thus the minimum thickness d satisfying our requirement is that for $\Delta\phi_{12} = \pi$ or $\dfrac{4 \times 1.5 \times d}{500\ nm} = 2$, giving $d = \dfrac{500\ nm}{3.0}$ or $\boxed{d = 166.7\ nm.}$

(b) Why are other parts of the visible spectrum not also preferentially transmitted? **ANSWER**: At this thickness, no *longer* wavelength (redder) light will have an optimal destructive interference of reflected light (preferential transmission); meanwhile, the next *shorter* wavelength (bluer) light to be preferential transmitted will have $\Delta\phi_{12} = 3\pi$, giving $\dfrac{4 \times 1.5 \times d}{\lambda_2} = 4 \implies \lambda_2 = 1.5d = 250.0\ nm$ [*i.e.* ultraviolet], which is not part of the visible spectrum.

(c) Will the transmission of any colors be sharply reduced? **ANSWER**: At this d, $\Delta\phi_{12} = 0$ (constructive interference $\implies$ maximum of reflection) if $\dfrac{4 \times 1.5 \times d}{\lambda_3} = 1 \implies \lambda_3 = 1000\ nm$ [infrared] and $\Delta\phi_{12} = 2\pi$ (ditto) if $\dfrac{4 \times 1.5 \times d}{\lambda_4} = 3 \implies \lambda_4 = 333.3\ nm$ [near ultraviolet].



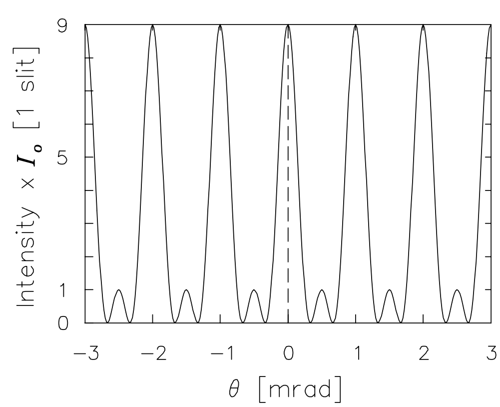
2. FRINGES IN A WEDGE: A pefectly flat piece of glass ($n = 1.45$) is placed over a perfectly flat piece of black plastic ($n = 1.30$) as shown below They touch at A. Green light of wavelength 525 nm is incident normally from above. Any light transmitted into the plastic is completely absorbed. The location of the dark fringes in the reflected light is shown in the sketch at lower right.

(a) How thick is the space between the glass and the plastic at B? **ANSWER**: Refer again to the sketch of *rays* in problem 1. In this case $n_A = 1.45$, $n_B = 1.0$ and $n_C = 1.30$, so ray 1 reflects off a *less* dense medium $\Longrightarrow$ no phase shift. Ray 2, however, reflects off a *denser* medium and so picks up a phase shift of π upon reflection; it also travels further than ray 1, so the net phase difference between rays 1 and 2 is $\Delta\phi_{12} = \pi\left(\frac{4dn_B}{\lambda} - 1\right)$ — *i.e.* just as in problem 1! Here as usual λ refers to the wavelength *in vacuum*; n_B is the index of refraction in the medium between the plates, in this case air. This gives destructive interference ($\Delta\phi_{12} = -\pi$) for $d = 0$, which explains the dark fringe where the plates touch at **A**. $\checkmark$ Between **A** and **B** we go through exactly 6 cycles of light and dark fringes ($6 \times 2\pi$ in phase shift) $\Longrightarrow \frac{4dn_B}{\lambda} = 12$ or $d = \frac{3 \times 525\ nm}{1.0}$ or $\boxed{d = 1575\ nm}$ at **B**.

(b) Water ($n = 1.33$) seeps into the region between the glass and plastic. How many dark fringes are seen when all the air has been displaced by water? **ANSWER**: Now *both* reflections (rays 1 and 2) are off *less dense* media $\Longrightarrow$ *neither* picks up an extra phase shift of π and we have $\Delta\phi_{12} = \pi\left(\frac{4dn_B}{\lambda}\right)$. There is now a *bright* fringe ($\Delta\phi_{12} = 0$) at **A** and at **B** we have $\Delta\phi_{12} = 2\pi \times \left(\frac{2 \times 1575 \times 1.33}{525}\right) = 7.98 \times (2\pi)$ — *i.e.* almost 8 full cycles of light and dark $\Longrightarrow$ another light fringe at **B** and $\boxed{\text{8 dark fringes in between.}}$

(The straightness and equal spacing of the fringes is an accurate test of the flatness of the glass.)

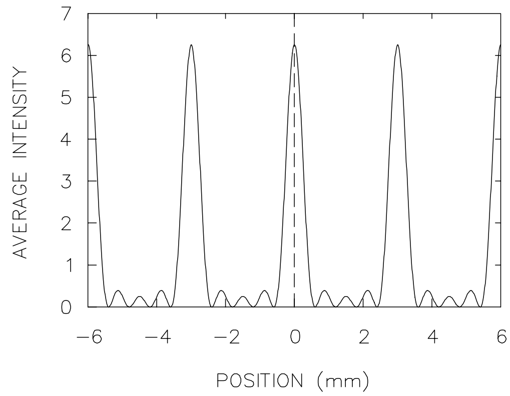
3. THREE-SLIT INTERFERENCE PATTERN: Light of wavelength 600 nm is incident normally on *three* parallel narrow slits separated by 0.60 mm. Sketch the intensity pattern observed on a distant screen as a function of angle θ for the range of values $-0.003 \le \theta \le 0.003$ radians.



ANSWER: We can use the formula $I = I_0\left[\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}}\right]^2$ where $\delta = 2\pi\left(\frac{d\sin\theta}{\lambda}\right)$ to draw the result by brute calculational effort, but it is more instructive (and a lot less effort!) to generate the sketch by a sequence of simpler qualitative arguments.

First consider the "gross structure" of the interference pattern: primary maxima occur when the phase difference between *adjacent slits* (separated by $d = 6.0 \times 10^{-4}\ m$) is an integer multiple of 2π: $\delta = 2\pi \Longrightarrow$ our old friend $d\sin\theta = m\lambda$. Here $\lambda = 6.0 \times 10^{-7}\ m$ so the criterion is $\sin\theta_m \approx \theta_m = m/1000$ *i.e.* after the central maximum at $\theta = 0$, we get <u>principle maxima every $10^{-3}\ rad \equiv 1\ mrad$</u> [*milliradian*]. This corresponds to a PHASOR diagram where all three phasors line up. There are *zeroes* for diagrams where $\delta = 2\pi\left(\frac{m \pm 1}{3}\right)$ giving $\left(\frac{m \pm 1}{3}\right)\lambda = d\sin\theta_z \approx d\theta_z$ — *i.e.* when $\theta_z = \left(\frac{m \pm 1}{3}\right)\ mrad$. Finally, when $\delta = (2m + 1)\pi$ the phasor diagram shows the first two slits π out of phase, exactly cancelling each other and leaving the third slit "by itself" to contribute an intensity equal to that of a single slit on its own: a <u>*secondary maximum*</u> whose intensity is $1/N^2 = 1/9$ of that in the principal maxima. The results are sketched at right above.

4. **N-SLIT INTERFERENCE PATTERN:** The figure below shows the intensity pattern produced by light passing through an opaque foil with N narrow slits 0.3 mm apart and falling on a screen parallel to the foil 2.0 m distant.



(Neglect the finite widths of the slits; this is an *interference* problem, *not* a *diffraction* problem.)

NOTE: The following derivation is far more verbose than necessary to solve the problem and is shown in detail merely to document the explanation given in class for the simple qualitative rules (number of minima and secondary maxima between principal maxima, *etc.*) that allow one to quickly analyze an interference pattern. All you really need to solve this problem are those rules and the simple criterion for a principal maximum: $d\sin\theta_m = m\lambda$.

(a) What wavelength of light is being used? **ANSWER**: Since $\tan\theta_{\max} = \dfrac{6\ mm}{2\ m} = 3\times10^{-3}$, we may use the small-angle approximation ($\sin\theta \approx \theta$), giving $\delta = \dfrac{2\pi d}{\lambda}\theta$ and so $I = I_0\left[\dfrac{\sin\left(N\pi\frac{d}{\lambda}\theta\right)}{\sin\left(\pi\frac{d}{\lambda}\theta\right)}\right]^2$. For convenience define $x \equiv \pi\dfrac{d}{\lambda}\theta \Longrightarrow \dfrac{I}{I_0} = \left[\dfrac{\sin(Nx)}{\sin x}\right]^2$, which has zeroes wherever $\sin Nx = 0$, *except* when $\sin x = 0$; in that case we use the rule $\lim\limits_{x\to0}\dfrac{\sin Nx}{\sin x} = \lim\limits_{x\to0}\dfrac{\frac{d}{dx}(\sin Nx)}{\frac{d}{dx}(\sin x)} = \lim\limits_{x\to0}\dfrac{N\cos Nx}{\cos x} = N$. Thus where $\sin x = 0$ (*i.e.* where $x = m\pi$ or where $\theta = m\lambda/d$) we get a *principal maximum* with $I = N^2 I_0$. We see such maxima every 3 *mm* at a distance of 2 *m*; *i.e.* since $\tan\theta \approx \theta$, every 1.5 *mrad* or $1.5\times10^{-3} = \dfrac{\lambda}{d}$
$\Longrightarrow \lambda = 1.5\times10^{-3}d = 1.5\times10^{-3}\times0.3\times10^{-3}\ m$ or $\boxed{\lambda = 4.5\times10^{-7}\ m = 450\ nm.}$

(b) How many slits are there? **ANSWER**: In between principal maxima we have $(N-1)$ zeroes where $\sin Nx = 0$ but $\sin x \neq 0$. For instance, between $x = 0$ and $x = \pi$ we have $x = \frac{\pi}{N},\ x = \frac{2\pi}{N},\ x = \frac{3\pi}{N},\ \dots\ x = (N-1)\frac{\pi}{N}$ all giving $Nx =$ a multiple of π and thus $\sin Nx = 0$. The general rule is thus $\boxed{(N-1)\text{ ZEROES and therefore }(N-2)\text{ SECONDARY MAXIMA \underline{between} principal maxima.}}$ We can therefore "read off the figure" $\boxed{N = 5\text{ slits.}}$