THE UNIVERSITY OF BRITISH COLUMBIA

Physics 401 Assignment # 5:
RELATIVISTIC ELECTRODYNAMICS

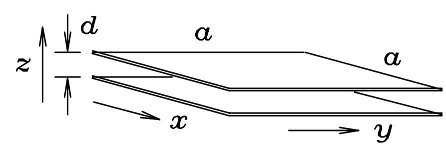
Wed. 1 Feb. 2006 — finish by Wed. 8 Feb.

This is a relatively [pun intended] short Assignment, since the first Midterm Exam is on Monday February 6 (in class, 50 minutes). Nevertheless it will count the same as other Assignments, if you choose to tackle it. For the exam, you may bring your own *1-page summary sheet* with any hard-to-remember equations *etc.* The exam will cover all of Chapters 7 and 8, sections 10.1 of Chapter 10, and all of Chapter 12. There will not be anything on the Midterm explicitly about $F^{\mu\nu}$ (second problem below) but applications of Eqs. (12.108) are fair game.

1. If we stay in the Lorentz gauge ($\partial_\mu A^\mu = 0$), A^μ is a genuine 4-vector and therefore $A_\mu A^\mu$ is a Lorentz scalar. Use this fact to show that, in *any* frame, $|\vec{\boldsymbol{J}}|^2 = c^2(\rho^2 - \rho_0^2)$, where ρ_0 is the charge density in the frame where $\vec{\boldsymbol{J}} = 0$.

2. Use the formal definition of the FIELD TENSOR $F^{\mu\nu}$ in Eq. (12.118)[1] and the rule for its Lorentz transformation,[2] $F'^{\mu\nu} = \Lambda^\mu_\alpha \Lambda^\nu_\beta F^{\alpha\beta}$, to derive the equations analogous to Eqs. (12.108) describing the transformation of $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ under a "boost" into a reference frame moving at velocity u (with the usual corresponding definitions of β and γ) in the positive $\hat{\boldsymbol{z}}$ direction.[3]

3. A capacitor made from two square parallel plates a on a side and d apart is given a charge $-Q$ on the upper plate and $+Q$ on the lower plate. Let the origin of coordinates be in the centre of the capacitor, the edges of the plates parallel to $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ and the gap in the $\hat{\boldsymbol{z}}$ direction.



Using Lorentz transformations, find $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ inside the capacitor

(a) in a frame moving at a velocity u in the $\hat{\boldsymbol{x}}$ direction;

(b) in a frame moving at a velocity u in the $\hat{\boldsymbol{z}}$ direction.

(c) For the former case, compare your result with what you would expect from simply transforming the dimensions of the plates into the moving frame and treating their motion as sheets of current.

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[1] Let's acquiesce to Griffiths' notation for this problem.

[2] The Einstein summation convention is assumed: sum over all repeated indices.

[3] Eqs. (12.108) are for a "boost" in the $\hat{\boldsymbol{x}}$ direction. This is rather tedious, especially since the result is rather obvious when you're done, but everyone should do it once in order to understand that the definition of $F^{\mu\nu}$ is more useful formally (for elegantly expressing the essence of electromagnetism) than for solving practical problems.