## BIOL/PHYS 438

# Zoological Physics

- Logistics
- Ch. 5: **"Animals in Motion"** wrapup
  - CentriXXXal forces, Terminal velocity, . . .
- Ch. 6: **"Locomotion"** wrapup
  - Swim, Fly, Walk . . .

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# Logistics

Assignment 1: Solutions now online!

Assignment 2: Solutions online soon!

Assignment 3: Solutions online soon!

Assignment 4: due **Thursday**

Jess will be away next week (5-9 March)

Hopefully your **Projects** are underway by now . . .

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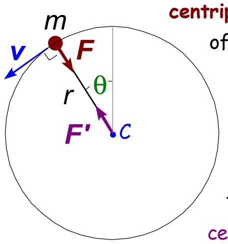
# Circular Motion review

Clarification: **centripetal** vs. "centrifugal" forces:

In order for a particle of mass $m$ to move at a constant speed $v$ in a circle of radius $r$ about a center $C$, it must be subjected to a **centripetal** ("toward the center") **force** of magnitude $F = mv^2/r = mr\omega^2$ (where $\omega \equiv d\theta/dt$) in order to cause the centripetal acceleration $a = v^2/r = r\omega^2$ implicit in uniform circular motion.



This implies an equal and opposite centrifugal force $F'$ acting on the center $C$ (**not** on the particle!).

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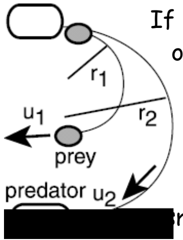
# Predator vs. Prey: Turning Radius

**Figure 5.7a** in the textbook suggests that the prey can turn more sharply than the predator (and thus escape) because it is **smaller**. This is not the case.



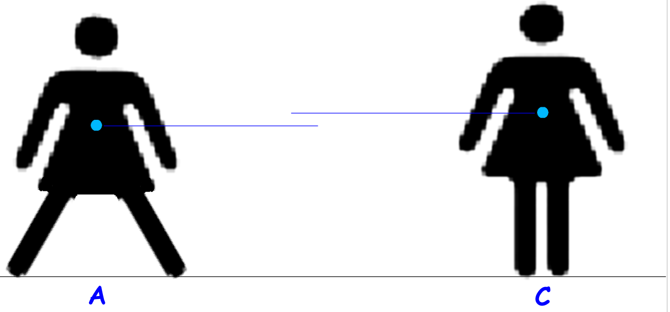
[redacted]

If both animals have the same coefficient of friction $\mu$ with the ground, then the largest centripetal force they can exert is $\mu Mg$, causing the same centripetal acceleration $a = u^2/r = \mu g$ for both animals. If the prey has a maller turning radius, it is because it is **slower** than the predator, not smaller.

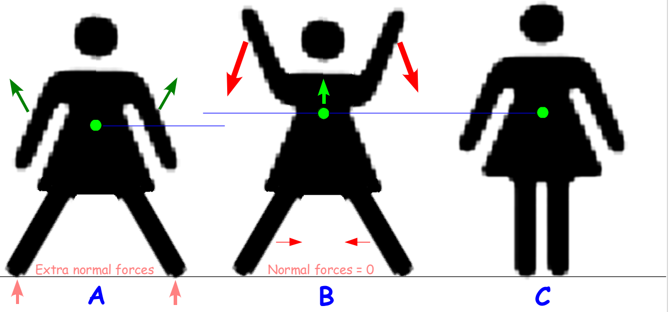
Lesson: **slow down** for "broken field running" !

# Crane Stance

A karate master stands in position **A**. How can she get to position **C** in one move without bending her knees or sliding her feet on the floor ?



**Answer**: first she quickly lifts her arms, raising her centre of gravity to the desired level and giving it a small upward momentum; then she abruptly reverses the motion and lowers her arms, lifting the rest of her body so that she can move her legs inward freely.



## Exam question (?)

Could a **heron** on the airless **Moon** get off the ground by flapping its wings (while holding its breath)? Explain!

# Allometrics: $X = aM^{\alpha}$

| parameter $X$ | factor $a$ | exponent $\alpha$ |
|---|---|---|
| body surface in $m^2$ | 0.11 | 0.65 |
| brain mass (man) in kg | 0.085 | 0.66 |
| brain mass (non primates) in kg | 0.01 | 0.7 |
| breathing frequency in Hz | 0.892 | – 0.26 |
| cost of transport (running) in J/m·k | 7 | – 0.33 |
| cost of transport(swimming) in J/m·kg | 0.6 | – 0.33 |
| effective lung volume in $m^3$ | $5.67 \cdot 10^{-5}$ | 1.03 |
| frequence of heartbeat in Hz | 4.02 | – 0.25 |
| heart mass in kg | $5.8 \cdot 10^{-3}$ | 0.97 |
| life time in years | 11.89 | 0.20 |
| metabolic rate in W | 4.1 | 0.75 |
| muscle mass in kg | 0.45 | 1.0 |
| skeletal mass (cetaceans) in kg | 0.137 | 1.02 |
| skeletal mass (terrestrial) in kg | 0.068 | 1.08 |
| speed of flying in m/s | 15 | 1/6 |
| speed of walking in m/s | 0.5 | 1/6 |

# Heartbeat Quota:

$$N_{hb} = 4.02\times11.89\times(365\times24\times60\times60)\ M^{-0.05}$$

$$= 1.5\times10^{9}\ M^{-0.05}$$

more or less independent of $M$.

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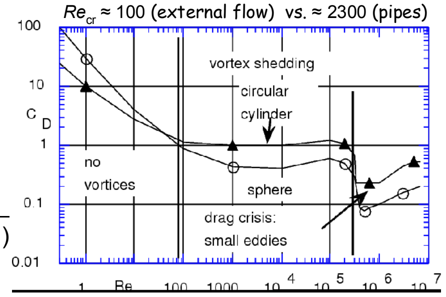
# Terminal Velocity -- when $Mg = F_{drag}$

Sphere: $\frac{4}{3}\pi R^3\rho g = F_{drag} = \underset{F_{Stokes}}{6\pi R\eta u} + \underset{F_{hydrodynamic}}{\frac{1}{2}\pi R^2 C_D \rho u^2}$

For $Re > 100$, $C_D \sim 1$; for $Re < 100$, $F_h \ll F_s$ so it does no harm to set $C_D = 1$. Then we can solve a quadratic equation for $u$:

$$u = (6\eta/R\rho)\times\left(\sqrt{1 + (R^3\rho g/27\eta^2)} - 1\right)$$

$Re_{cr} \approx 100$ (external flow) vs. $\approx 2300$ (pipes)



$$Re = 2uR/\nu = 2uR\rho/\eta$$

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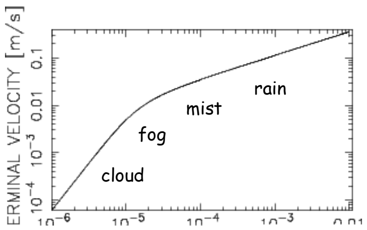
# Raindrop Terminal Velocity

$$u = (6\eta/R\rho)\times\left(\sqrt{1 + (R^3\rho g/27\eta^2)} - 1\right)$$

with $g = 9.81$, $\rho = 10^3$ and $\eta = 1.8\times10^{-5}$ gives

$$u = (1.08\times10^{-7}/R)\times\left(\sqrt{1 + 1.12\times10^{15} R^3} - 1\right):$$



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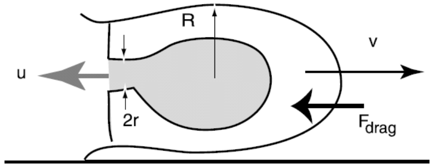
# Jet Propulsion

Bernoulli: $\Delta p = \frac{1}{2}\rho u^2$ mass flow: $J = \pi r^2 \rho u$

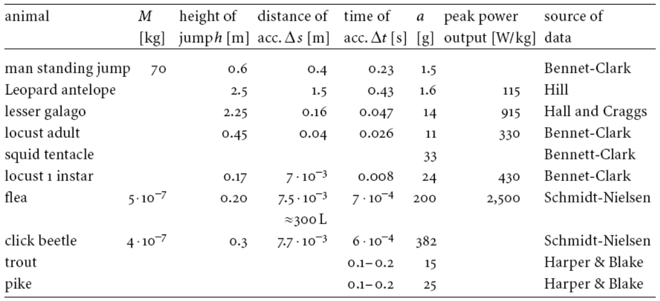
Thrust: $F = \pi r^2 \Delta p = \frac{1}{2}\pi r^2 \rho u^2 = Ju = J^2/\pi r^2 \rho$

There's a factor of 2 discrepancy . . . .



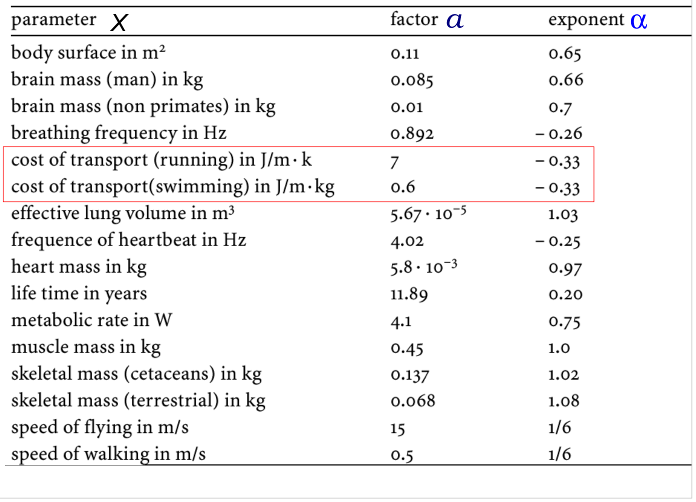
# High Jumpers

**Table 5.6.** Jumping performance of animals of body mass $M$. Height $h$, distance $\Delta s$, acceleration $a$, and peak power



| animal | $M$ [kg] | height of jump $h$ [m] | distance of acc. $\Delta s$ [m] | time of acc. $\Delta t$ [s] | $a$ [g] | peak power output [W/kg] | source of data |
|---|---|---|---|---|---|---|---|
| man standing jump | 70 | 0.6 | 0.4 | 0.23 | 1.5 | | Bennet-Clark |
| Leopard antelope | | 2.5 | 1.5 | 0.43 | 1.6 | 115 | Hill |
| lesser galago | | 2.25 | 0.16 | 0.047 | 14 | 915 | Hall and Craggs |
| locust adult | | 0.45 | 0.04 | 0.026 | 11 | 330 | Bennet-Clark |
| squid tentacle | | | | | 33 | | Bennett-Clark |
| locust 1 instar | | 0.17 | $7 \cdot 10^{-3}$ | 0.008 | 24 | 430 | Bennet-Clark |
| flea | $5 \cdot 10^{-7}$ | 0.20 | $7.5 \cdot 10^{-3}$<br>≈300 L | $7 \cdot 10^{-4}$ | 200 | 2,500 | Schmidt-Nielsen |
| click beetle | $4 \cdot 10^{-7}$ | 0.3 | $7.7 \cdot 10^{-3}$ | $6 \cdot 10^{-4}$ | 382 | | Schmidt-Nielsen |
| trout | | | | 0.1–0.2 | 15 | | Harper & Blake |
| pike | | | | 0.1–0.2 | 25 | | Harper & Blake |

# Allometrics: $X = aM^{\alpha}$



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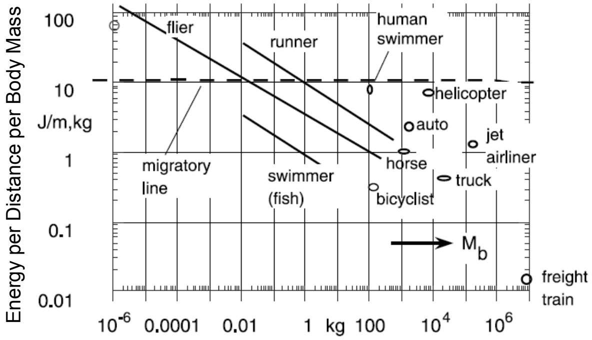
# Cost of Transport



# Fish Can't Swim? (Gray's paradox)

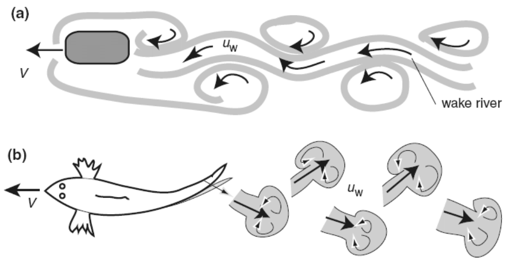
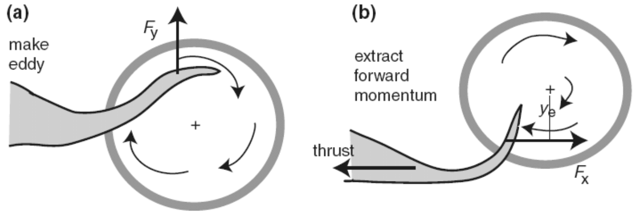


**Fig. 6.11.** (**a**) Von Karman vortex street of wakes of a dragged object (passive wake). (**b**) Active wake of fish.

# Kicking off the Eddies

(what goes around, comes around)



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# Swimmers

Speed $U_{sw} \approx 0.5\, M^{0.19}$ Length $L_{large} \approx 1.5\, M^{0.43}$

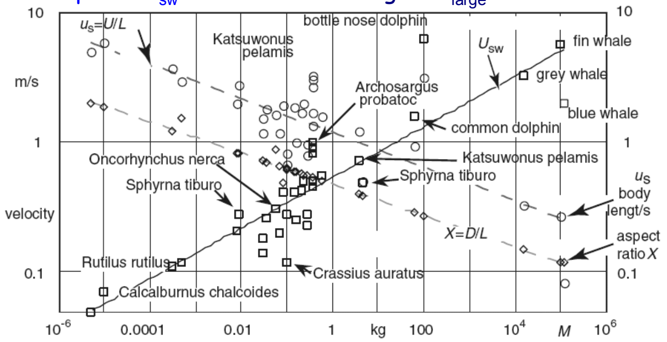

Fig. 6.7. Swimming velocity $U_{sw}$ given in m/s and in body length per sec ($U_{sw}/L$); and body aspect ratios $X = D/L$ as function of body mass $M$. Data collected from various sources by M. Y. Yeung [2001]. The gray whale velocity was measured by Megill [2000]

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# Fliers

Speed $U \approx 15\, M^{1/6}$ Metabolic velocity $U_m \approx 40\, M^{-1/4}$

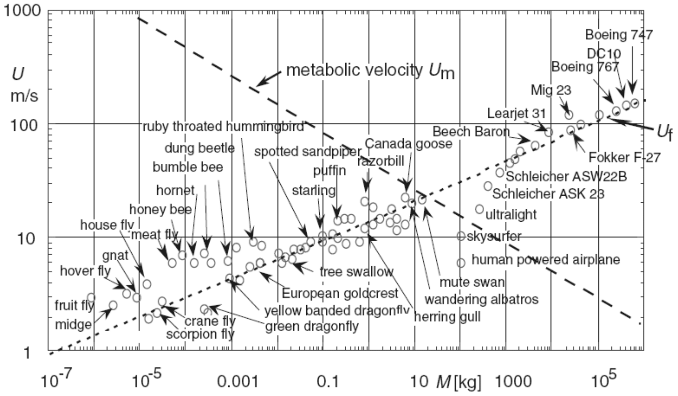

Fig. 6.20. The great flight diagram. Data adopted from Tennekes [1997]

# Runners

Endurance running speed $U_{er} \approx C_r M^{-0.023} \approx$ const.

**Table 6.1.** Mass $M$, impact force $F_{max}$, leg length $L$, and measured speeds $U_{exp}$ reported by Farley et al. [1993] and derived parameters: resonance velocity $U_r$, vertical acceleration $a_{vert}$, metabolic endurance running constant $C_r$, and maximum tension leg length scaling constant $C_2$

| animal | $M$ [kg] | $F_{max}$ [N] | $L$ [m] | $U_{exp}$ [m/s] | $U_r$ [m/s] | $A_{vert}$ [m/s²] | $C_r$ | $C_2$ |
|---|---|---|---|---|---|---|---|---|
| kangaroo rat | 0.112 | 6.0 | 0.099 | 1.80 | 1.84 | 53.6 | 1.71 | 0.205 |
| white rat | 0.144 | 3.0 | 0.065 | 1.10 | 0.93 | 20.8 | 1.05 | 0.124 |
| wallaby | 6.86 | 300 | 0.330 | 3.00 | 3.04 | 43.7 | 3.14 | 0.174 |
| dog | 23.6 | 500 | 0.500 | 2.80 | 2.60 | 21.2 | 3.01 | 0.174 |
| goat | 25.1 | 500 | 0.480 | 2.80 | 2.47 | 19.9 | 3.02 | 0.164 |
| red kangaroo | 46.1 | 2000 | 0.580 | 3.80 | 4.01 | 43.4 | 4.15 | 0.162 |
| horse | 135 | 3000 | 0.750 | 2.90 | 3.27 | 22.2 | 3.27 | 0.146 |
| | | | | | | average | 2.76 | 0.16 |
| | | | | | | standard dev. | | ±0.01 |

# Limits