the University of British Columbia

Physics 108

SOLUTIONS

FIRST MIDTERM - 04 February 2005

Time: 50 minutes

Instructor(s): Jess H. Brewer

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1. QUIKIES [10 marks each — 60 total]

(*a*) Under what circumstances would the *entropy* of a system *decrease* with the *addition of energy*, and what could you say about the *temperature* of such a system? [1]

(*b*) Charges of $+2Q$ and $-Q$ are located in the plane of the page as shown. *Sketch the region* in the same plane (if any) where the resultant *electric field* is *zero*. [2]

$+2Q$ $\qquad\qquad\qquad$ $-Q$ $\qquad\qquad\qquad$ $\Downarrow$

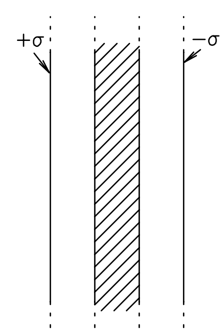
$\bullet$ $\qquad\qquad\qquad\qquad$ $\bullet$ $\qquad\qquad\qquad\qquad$ $\bullet$

(*c*) In broad general terms, explain why the thermal distribution of particle speeds is **not** the same in a **1**-dimensional ideal gas as it is in a **3**-dimensional ideal gas of the same particles at the same temperature. [3]

(*d*) A positive point charge Q is fixed at an arbitrary location (*not* on the axis) inside an uncharged, thin-walled copper tube whose length L is much larger than its radius R. The charge is not located near either end. Define r as the perpendicular distance from the axis of the tube. Match up **all** the left and right side phrases that make up *true* sentences: [4]

The electric field inside the tube ($r < R$)	is zero.
	is a complicated function of the charge's position.
The electric field outside the tube ($r > R$)	has a magnitude $E \approx Q/2\pi\epsilon_\circ Lr$ except near the ends.
	is in the $\hat{\boldsymbol{r}}$ direction.

(*e*) The diagram shows an edge-on view of an electrically neutral, semi-infinite, flat conducting slab with a parallel sheet of uniformly distributed positive charge (charge per unit area $+\sigma_\circ$) on the left and a parallel sheet of uniformly distributed negative charge (charge per unit area $-\sigma_\circ$) on the right. What is the direction and magnitude of the electric field ...



i) ... to the left of the positive sheet of charge?

ii) ... between the positive sheet of charge and the slab?

iii) ... inside the slab?

iv) ... between the slab and the negative sheet of charge?

v) ... to the right of the negative sheet of charge?

Answers: [5] [6] [7]

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[1] There must be a *limit* to the amount of energy the system can *hold*, otherwise more energy is bound to offer more possibilities for redistribution, and thus more entropy. If the entropy is decreasing with increasing energy (*i.e.* the derivative is negative) then by definition the *temperature* is *negative* (*i.e. hotter than infinite temperature.*)

[2] There is only one point where the electric field is zero: exactly as far to the right of the negative charge as the separation between the charges.

[3] The probability of a given single particle state of a given energy being occupied is the same for both, but the *density of states* (*i.e.* the number of possible states with speeds within a given dv of v) is different because there are more *directions* for the vector velocity to point in 3 dimensions than in 1 dimension. You can also describe this in terms of modes of standing waves, but the classical explanation is adequate for full credit.

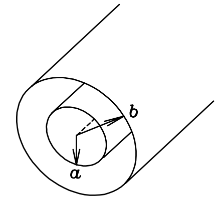
[4] The electric field inside the tube ($r < R$) is a complicated function of the charge's position. The electric field outside the tube ($r > R$) has a magnitude $E \approx Q/2\pi\epsilon_\circ Lr$ (except near the ends) and is in the $\hat{\boldsymbol{r}}$ direction.

[5] The electric field inside any conductor is zero.

[6] To the left of the positive sheet of charge and to the right of the negative sheet of charge, the electric field is zero, because a Gaussian "pillbox" surface cutting through the entire array at right angles encloses no net charge.

[7] A negative surface charge is attracted to the left side of the conductor by the positive sheet of charge, leaving behind an equal and opposite positive surface charge on the right side. These induced surface charges must cancel the electric

(*f*) Referring to the previous diagram, calculate the induced surface charge σ per unit area on each side of the slab in terms of $\sigma_\circ$. [8]



2. CHARGED COAXIAL CONDUCTORS [40 marks]

A long copper cylinder of radius a is surrounded by a coaxial copper tube whose inner radius is b, as shown. The inner cylinder carries a uniform charge per unit length (λ) and the outer shell has an equal and opposite charge per unit length ($-\lambda$) so that the system as a whole is electrically neutral.

(*a*) [5 marks] If r is the distance from the axis, what is the electric field for $r < a$? Explain. [9]

(*b*) [5 marks] What is the electric field $\vec{\boldsymbol{E}}(r)$ for $r > b$? Explain. [10]

(*c*) [10 marks] What is the electric field $\vec{\boldsymbol{E}}(r)$ between the two cylinders ($a < r < b$)? [11]

Now consider the case where $a = 1$ m, $b = 1.01$ m and $\lambda = +10^{-10}$ C/m. Since $(b - a) \ll a$, you can treat the electric field between the cylinders as approximately constant in magnitude. The 1 cm gap between the inner cylinder and the outer tube is evacuated except for 100 tiny beads, each of which contains a single excess electron fixed at its centre so that its net charge is $-e$. The beads stick to the copper surfaces, but are occasionally shaken loose by thermal motion. The whole system is in thermal equilibrium at 300 K.

(*a*) [5 marks] What is the difference $\varepsilon = U(b) - U(a)$ between the electrostatic potential energy $U(b)$ of a bead stuck to the surface of the outer shell and that of a bead stuck to the surface of the inner cylinder, $U(a)$? [12]

(*b*) [15 marks] *On average*, how many beads are stuck to each surface? [13]

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fields of the sheets inside the conductor, so they produce electric fields of their own with the same magnitude as those of the sheets, and (between the sheets and the conductor) in the same direction. So in both gaps we get an electric field to the right whose (uniform) magnitude is $\boxed{E_{\text{gap}} = \sigma_\circ/\epsilon_\circ}$, twice that from a single sheet, $E_\circ = \sigma_\circ/2\epsilon_\circ$.

[8] There are several ways to think about this question; the trick is not to get them mixed up. The reasoning given in the answer to the previous question treats all sheets of charge (original or induced) on an equal basis, explaining the zero electric field within the conductor explicitly in terms of the sum of several contributions. This immediately gives $\boxed{|\sigma| = \sigma_\circ}$ on both sides of the slab. You can also use the general rule (obtained by taking a very small Gaussian "pillbox" that only encloses one surface of the conductor) that $E = \sigma/\epsilon_\circ$ near the surface of any conductor; in this case $E_{\text{gap}} = \sigma/\epsilon_\circ$ but since $E_{\text{gap}} = 2E_\circ$ and $E_\circ = \sigma_\circ/2\epsilon_\circ$ we get $\sigma/\epsilon_\circ = 2\sigma_\circ/2\epsilon_\circ$ or $|\sigma| = \sigma_\circ$ again.

[9] $\boxed{\text{Zero}}$. There is no net charge enclosed within a coaxial Gaussian cylinder of radius $r < a$.

[10] $\boxed{\text{Zero}}$. There is a positive charge $\lambda\ell$ from the inner conductor and a negative charge $-\lambda\ell$ from the outer conductor both enclosed within a coaxial Gaussian cylinder of length ℓ and radius $r > b$, but there is still no *net* charge enclosed.

[11] Now it gets interesting. A coaxial Gaussian cylinder of length *ell* and radius r between a and b encloses a net positive charge $Q = \ell\lambda$ which must equal $\epsilon_\circ E$ times the surface area $2\pi r\ell$ through which $\vec{\boldsymbol{E}}$ emerges, giving the familiar $\boxed{E(r) = \lambda/2\pi\epsilon_\circ r}$. The negative shell of charge at $r = b$ is not enclosed, so it contributes nothing to E.

[12] You are welcome to do an integral if you wish, but I excused you from this chore by specifying that you can treat E as constant in the narrow gap, giving simply $\varepsilon = eE(b-a)$. Check the sign: the negatively charged electron experiences an *inward* force so "out" is "uphill" and $U(b)$ is greater than $U(a)$. With $E \approx E(a) = \lambda/2\pi\epsilon_\circ a$ this gives $\varepsilon = e\lambda(b-a)/2\pi\epsilon_\circ a$. Putting in the numbers gives $\varepsilon = \frac{(1.602\times10^{-19})(10^{-10})(0.01)}{(2\pi)(0.8854\times10^{-11})(1)}$ or $\boxed{\varepsilon = 0.288 \times 10^{-20} \text{ J}}$.

[13] Here we have two states of different energies and therefore different Boltzmann factors. If we define the energy of a bead stuck to the surface at $r = a$ to be zero, then the ratio of the probability P_b of being stuck to the surface at $r = b$ to the probability P_a of being stuck to the surface at $r = a$ is $\exp(-\varepsilon/k_{\text{B}}T)$, where $T = 300$ K and $k_{\text{B}} = 1.3807\times10^{-23}$ J/K, giving $\varepsilon/k_{\text{B}}T = \frac{0.288\times10^{-20}}{0.4142\times10^{-20}} = 0.6953$ and $P_b/P_a = e^{-0.6953} = 0.499 \approx 1/2$. Since $P_a + P_b = 1 \approx (3/2)P_a$, $P_a \approx 2/3$ and $P_b \approx 1/3$, so on average $\boxed{67}$ of the 100 beads will be stuck to the surface at $r = a$ and $\boxed{33}$ will be stuck to the surface at $r = b$.