The University of British Columbia

# Physics 401 Assignment # 7:
# WAVES IN MEDIA

Wed. 22 Feb. 2006 — finish by Wed. 1 Mar.

**1.** (p. 395, Problem **9.18**) — **Practical Questions:**[1]

(*a*) Suppose you embedded some free charge in a piece of glass. About how long would it take for the charge to flow to the surface?

(*b*) Silver is an excellent conductor, but it's expensive. Suppose you were designing a microwave experiment to operate at a frequency of $10^{10}$ Hz. How thick would you make the silver coatings?

(*c*) Find the wavelength and propagation speed in copper for radio waves at 1 MHz. Compare the corresponding values in air (or vacuum).

**2.** (p. 396, Problem **9.19**) — **Skin Depth:**

(*a*) Show that the skin depth in a poor conductor ($\sigma \ll \omega\epsilon$) is $(2/\sigma)\sqrt{\epsilon/\mu}$ (independent of frequency). Find the skin depth (in meters) for (pure) water.[2]

(*b*) Show that the skin depth in a good conductor ($\sigma \gg \omega\epsilon$) is $\lambda/2\pi$ (where $\lambda$ is the wavelength *in the conductor*. Find the skin depth (in nanometers) for a typical metal [$\sigma \approx 10^7\ (\Omega\,\mathrm{m})^{-1}$] in the visible range ($\omega \approx 10^{15}\ \mathrm{s}^{-1}$), assuming $\epsilon \approx \epsilon_0$ and $\mu \approx \mu_0$. Why are metals opaque?

(*c*) Show that in a good conductor the magnetic field lags the electric field by 45°, and find the ratio of their amplitudes. For a numerical example, use the "typical metal" in the previous question.

**3.** (p. 398, Problem **9.21**) — **Silver Mirror:** Calculate the reflection coefficient for light at an air-to-silver interface [$\mu_1 = \mu_2 = \mu_0,\ \epsilon_1 = \epsilon_0,\ \sigma = 6 \times 10^7\ (\Omega\,\mathrm{m})^{-1}$], at optical frequencies ($\omega = 4 \times 10^{15}\ \mathrm{s}^{-1}$).

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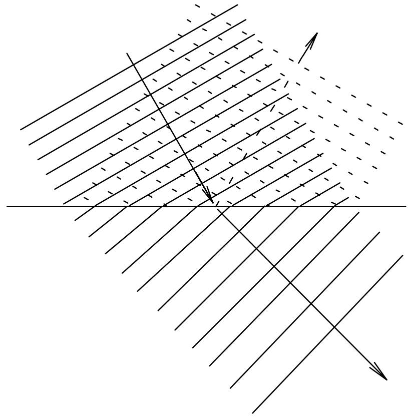
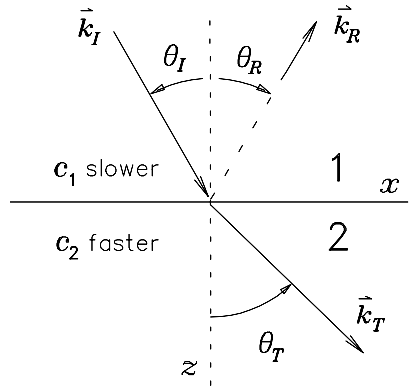
[1] Take $n = 1.5$ as the index of refraction typical of glass. You can look up electric permeabilities in Table 4.2 on page 180, magnetic susceptibilities in Table 6.1 on page 275 and resistivities in Table 7.1 on page 286.

[2] Your *calculation* should yield a skin depth of about $1.2 \times 10^4$ m for pure water. (No points for just the answer!)






**4.** (p. 413, Problem **9.37**) — **TIR:**

According to SNELL'S LAW, when light passes from an optically dense medium into a less dense one ($n_1 > n_2$) the propagation vector $\vec{\boldsymbol{k}}$ bends *away* from the normal (see Figure). In particular, if the light is incident at the **critical angle** $\theta_C \equiv \sin^{-1}(n_2/n_1)$, then $\theta_T = 90°$, and the transmitted ray just grazes the surface. If $\theta_I$ *exceeds* $\theta_C$, there is no refracted ray at all, only a reflected one. This is the phenomenon of **total internal reflection**,[3] on which light pipes and fiber optics are based. But the *fields* are not zero in medium 2; what we get is a so-called **evanescent wave**, which is rapidly attenuated and transports no energy into medium 2.[4]

A quick way to construct the evanescent wave is simply to quote the results of Sect. 9.3.3, with $k_T = \omega n_2/c$ and $\vec{\boldsymbol{k}}_T = k_T(\sin\theta_T\,\hat{\boldsymbol{x}} + \cos\theta_T\,\hat{\boldsymbol{z}})$; the only change is that $\sin\theta_T = (n_1/n_2)\sin\theta_I$ is now greater than 1, and so $\cos\theta_T = \sqrt{1-\sin^2\theta_T}$ is imaginary. (Obviously, $\theta_T$ can no longer be interpreted as an *angle*!)

($a$) Show that $\tilde{\vec{\boldsymbol{E}}}_T(\vec{\boldsymbol{r}},t) = \tilde{\vec{\boldsymbol{E}}}_{0_T}\,e^{-\kappa z}e^{i(kx-\omega t)}$, where $\kappa \equiv \dfrac{\omega}{c}\sqrt{(n_1\sin\theta_I)^2 - n_2^2}$ and $k \equiv \dfrac{\omega n_1}{c}\sin\theta_I$. This describes a wave propagating in the $x$ direction (*parallel* to the interface!) and attenuated in the $z$ direction.

($b$) Noting that $\alpha \equiv \dfrac{\cos\theta_T}{\cos\theta_I}$ is now imaginary, use Eqs. (9.109),

$$\textbf{TM:}\quad \tilde{E}_{0_R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0_I}\,, \qquad \tilde{E}_{0_T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0_I}\,,$$

to calculate the reflection coefficient for polarization parallel to the plane of incidence. [Notice that you get 100% reflection, which is better than at a conducting surface (see for example Problem **9.21**).]

($c$) Do the same for polarization perpendicular to the plane of incidence (use the results of Problem 9.16):

$$\textbf{TE:}\quad \tilde{E}_{0_R} = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)\tilde{E}_{0_I}\,, \qquad \tilde{E}_{0_T} = \left(\frac{2}{1+\alpha\beta}\right)\tilde{E}_{0_I}$$

($d$) In the case of polarization perpendicular to the plane of incidence, show that the (real) evanescent fields are $\vec{\boldsymbol{E}}(\vec{\boldsymbol{r}},t) = E_0\,e^{-\kappa z}\cos(kx-\omega t)\,\hat{\boldsymbol{y}}$, $\vec{\boldsymbol{B}}(\vec{\boldsymbol{r}},t) = (E_0/\omega)\,e^{-\kappa z}[\kappa\sin(kx-\omega t)\,\hat{\boldsymbol{x}} + k\cos(kx-\omega t)\,\hat{\boldsymbol{z}}]$.

($e$) Check that the fields in the last part satisfy all of MAXWELL'S EQUATIONS (9.67).

($f$) For those same fields, construct the Poynting vector and show that, on average, no energy is transmitted in the $z$ direction.

[3]Our present Provost, Lorne Whitehead, had an idea to build hollow-core light pipes using total internal reflection when he was a graduate student in our Department; he patented the idea, formed a company called (you guessed it) *TIR Ltd.* and built it up over a decade into a model of Canadian entrepreneurship, after which he came back to UBC as a Physics Professor, and the rest is, as they say, history.

[4]The evanescent fields can be detected by placing a second interface a short distance into medium 2; in a close analogue to quantum mechanical **tunneling**, the wave crosses the gap and reassembles beyond it. See F. Albiol, S. Navas and M.V. Andres, *Am. J. Phys.* **61**, 165 (1993). [You might wonder whether medium 3 needs to have $n_3 > n_2$ for this to work, or if any meaningful interface will do, or if there is a more subtle criterion. But you don't have to work that out for this problem!]