“SOLVING” QUADRATIC EQUATIONS

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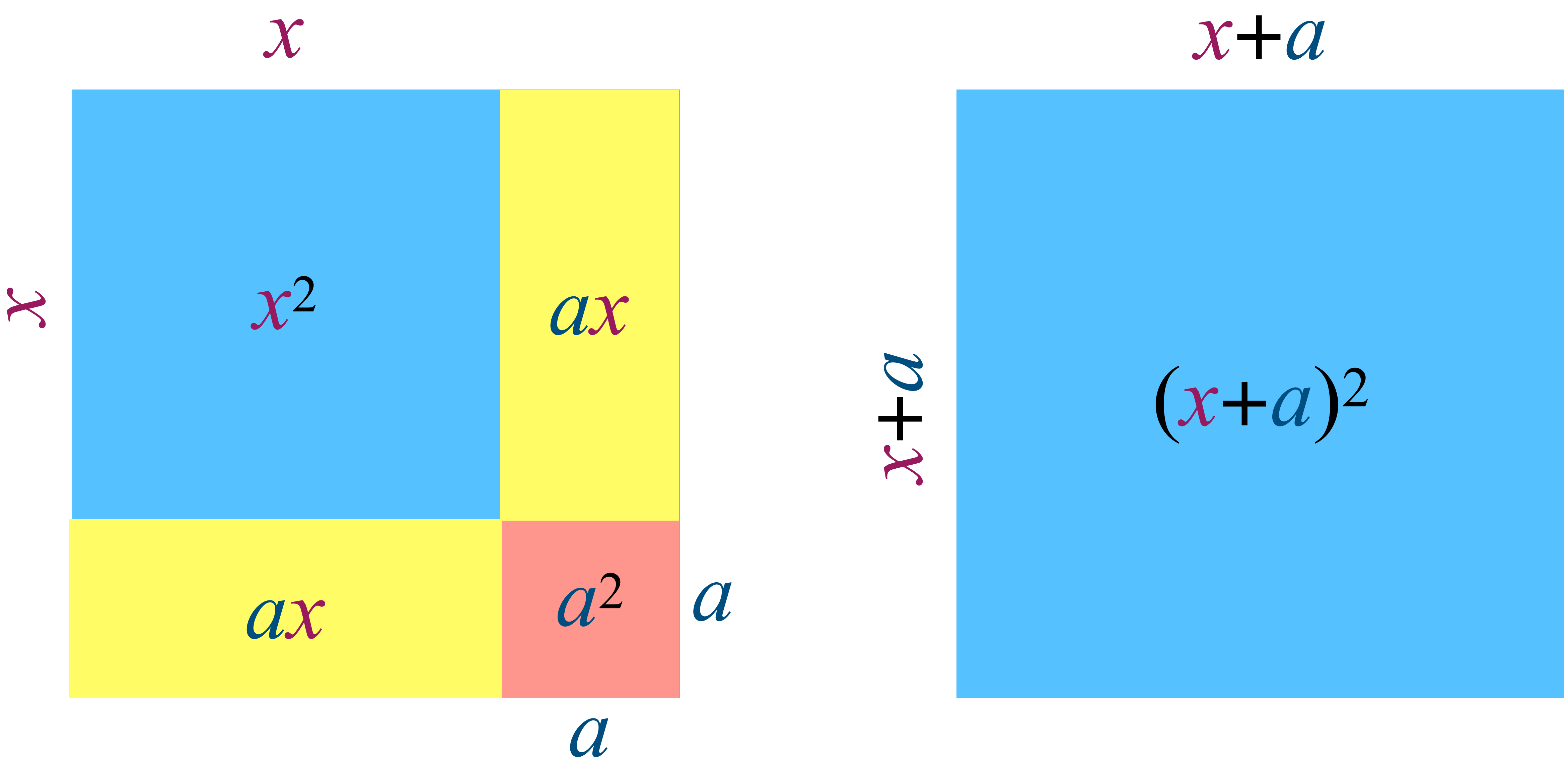
A "**solution**" is a formula with x on the left *by itself* and a function involving *only* a, b & c on the right.

But first, a little practice…

Geometrical Tricks with AREAS:



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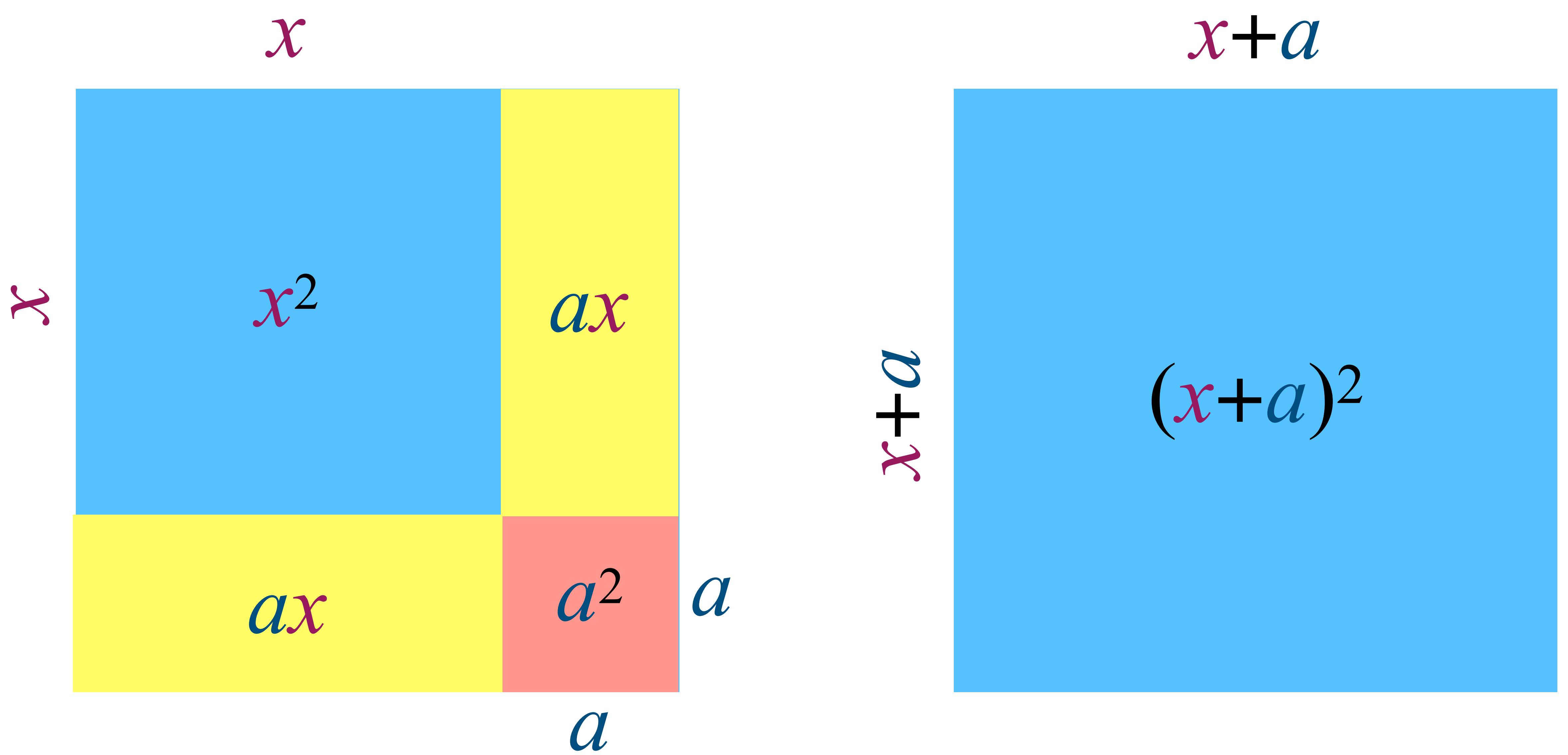

$$(x+a)^2 = x^2 + 2ax + a^2$$

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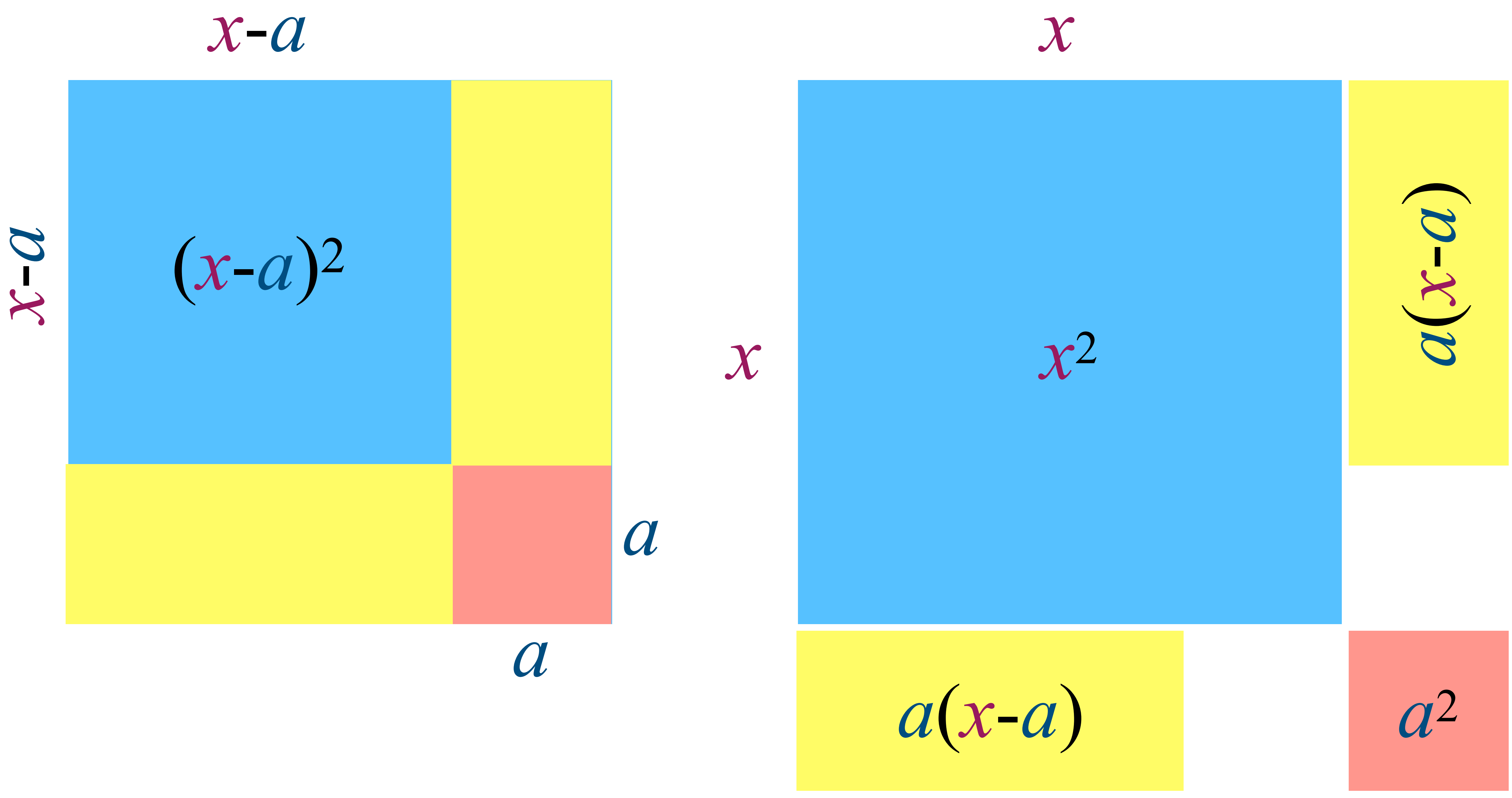

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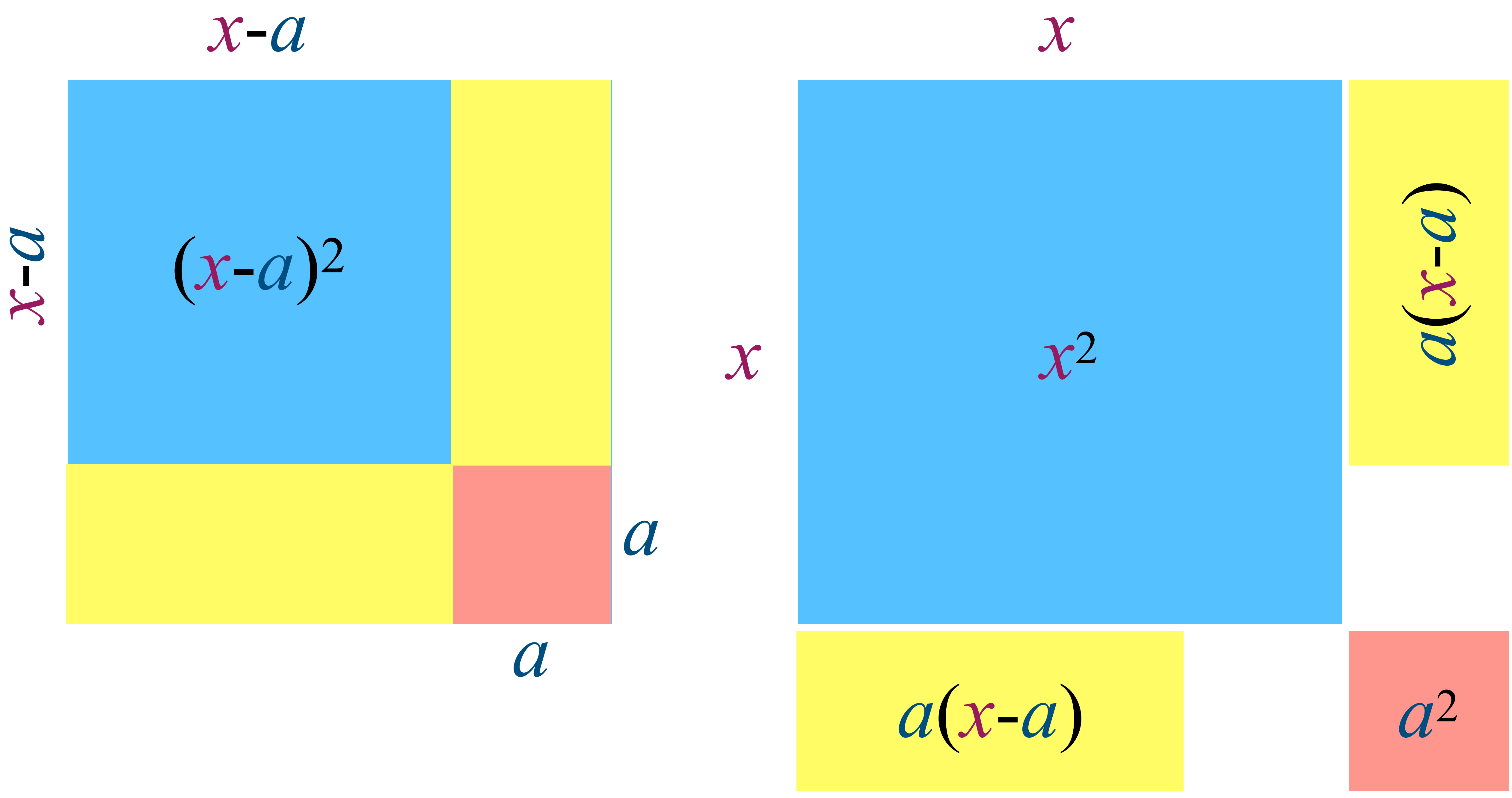

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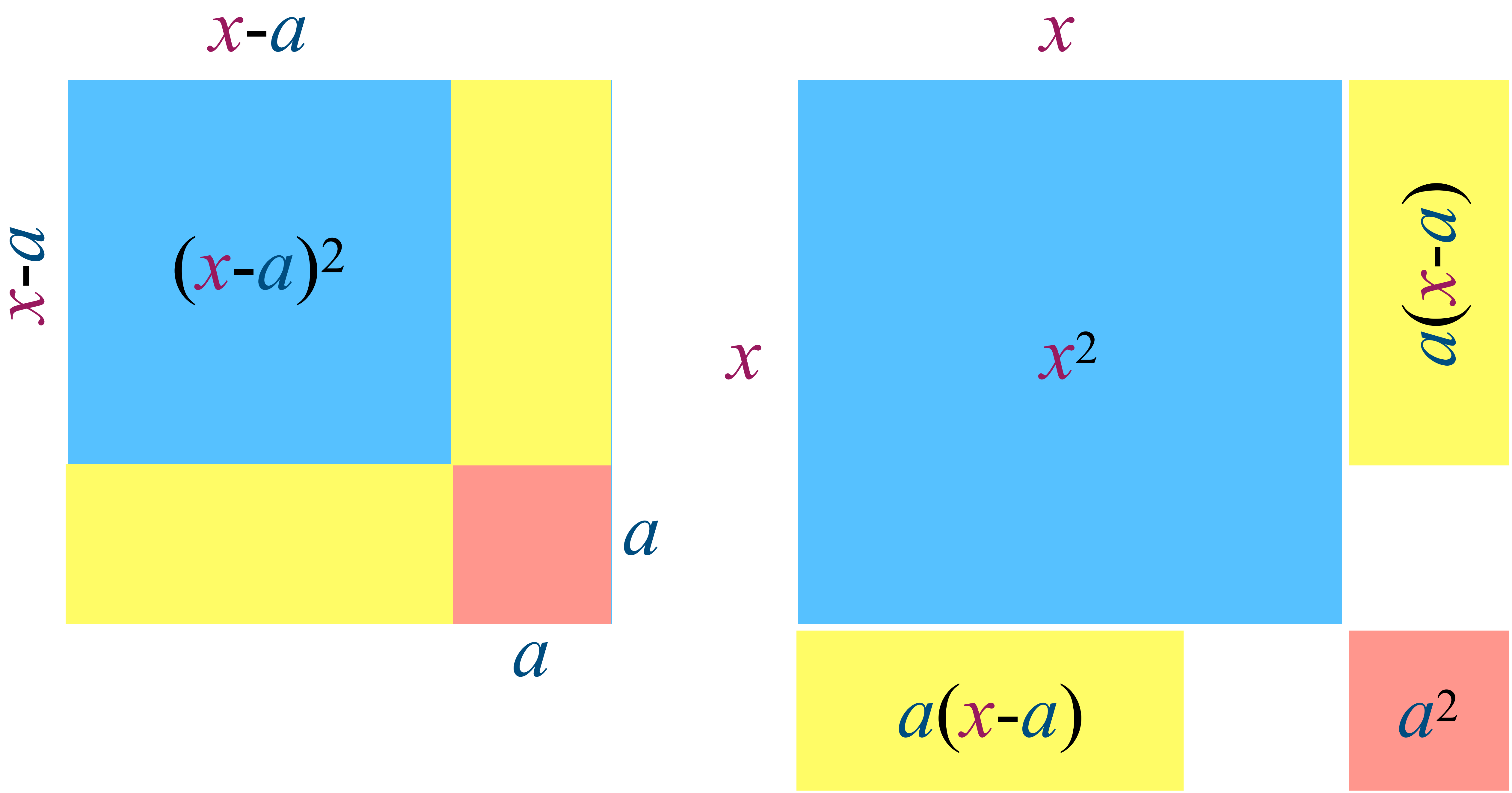

$$(x-a)^2 = x^2 - 2a(x-a) - a^2 = x^2 - 2ax + a^2$$

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Now, what can we add to both sides that will make the left side the *square* of something?

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compare

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$$(x+b/2a)^2 - (b/2a)^2 = - c/a$$

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$$x + b/2a = \pm\sqrt{(b/2a)^2 - c/a}$$

or

$$x = -b/2a \pm \sqrt{(b/2a)^2 - c/a}$$

$$(x+b/2a)^2 = (b/2a)^2 - c/a$$

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$$x + b/2a = \pm\sqrt{(b/2a)^2 - c/a}$$

or

$$x = - b/2a \pm \sqrt{(b/2a)^2 - c/a}$$

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$$x = \frac{- b \pm \sqrt{b^2 - 4ac}}{2a}$$

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The Quadratic Theorem

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✔

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$$x = \frac{-1 \pm \sqrt{1 - 2}}{2} = \frac{-1 \pm \boldsymbol{i}}{2}$$

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If $b^2 - 4ac$ is *negative*, the roots are ***complex***.