# “SOLVING” QUADRATIC EQUATIONS

of the form

$$ax^2 + bx + c = 0$$

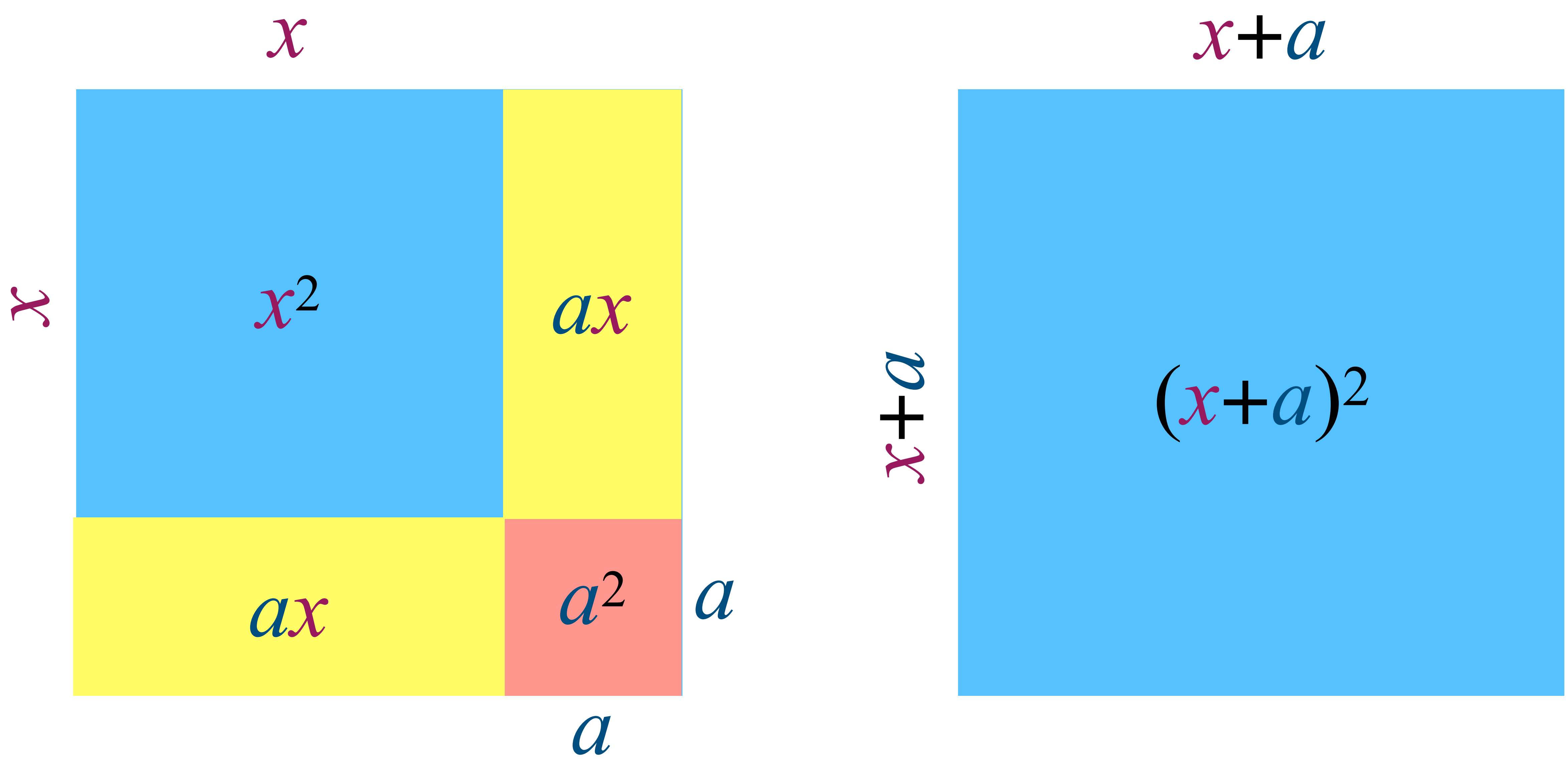
where $x$ is the “unknown”

and $a$, $b$ & $c$ are “constants”

A “**solution**” is a formula with $x$ on the left *by itself* and a function involving *only* $a$, $b$ & $c$ on the right.

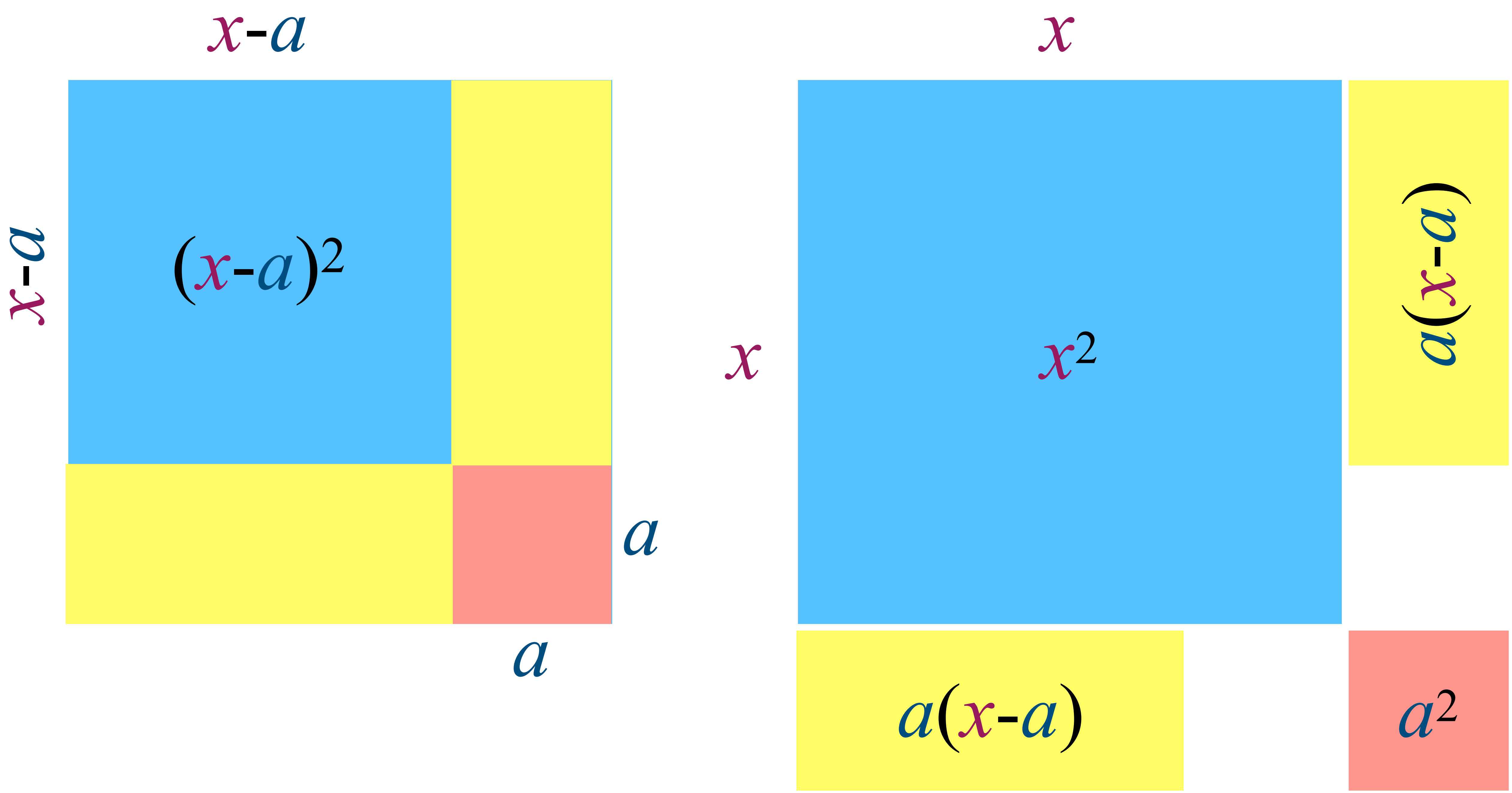
# But first, a little practice…

# Geometrical Tricks with AREAS:




$$(x+a)^2 = x^2 + 2ax + a^2$$

# Geometrical Trick with AREAS:




$$(x-a)^2 = x^2 - 2a(x-a) - a^2 = x^2 - 2ax + a^2$$

# Same Thing with just Algebra:

$$(x\text{-}a)^2 \equiv (x\text{-}a)(x\text{-}a)$$ (definition of "squared")

$$= x(x\text{-}a) - a(x\text{-}a)$$ (expand)

$$= x^2 - xa - (ax - a^2)$$ (distributive law)

$$= x^2 - 2ax + a^2$$

# Back to the Quadratic Equation:

$$ax^2 + bx + c = 0$$

$$- (c = c)$$

$$ax^2 + bx = -c$$

$$\div (a = a)$$

$$x^2 + (b/a)x = -c/a$$

Now, what can we add to both sides that will make the left side the *square* of something?

$x^2 + (b/a)x$ :

compare

$$(x+d)^2 = x^2 + 2dx + d^2$$

or $(x+d)^2 - d^2 = x^2 + 2dx$

so if $2d = b/a$, or $d = b/2a$,

$$x^2 + (b/a)x = (x+b/2a)^2 - (b/2a)^2$$

and $x^2 + (b/a)x = - c/a$ becomes

$$(x+b/2a)^2 - (b/2a)^2 = - c/a$$

or $(x+b/2a)^2 = (b/2a)^2 - c/a$

$$(x+b/2a)^2 = (b/2a)^2 - c/a$$

Take square root of both sides:

$$x + b/2a = \pm\sqrt{(b/2a)^2 - c/a}$$

or

$$x = -b/2a \pm \sqrt{(b/2a)^2 - c/a}$$

or

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Quadratic Theorem

# EXAMPLE:

$$x^2 + 2x + 1 = 0$$

$$a = 1,\ b = 2\ \&\ c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4 - 4}}{2} = -1$$

# EASY CHECK:

$$x^2 + 2x + 1 = 0$$

$$= (x + 1)^2$$

$$x + 1 = 0$$

$$x = -1$$

✔

# EXAMPLE:

$$x^2 + 3x + 2 = 0$$

$$a = 1,\ b = 3\ \&\ c = 2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{9 - 8}}{2} = -1 \text{ or } -2$$

# EXAMPLE:

$$x^2 + x + \tfrac{1}{2} = 0$$

$$a = 1, \quad b = 1 \quad \& \quad c = \tfrac{1}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1 - 2}}{2} \qquad = \frac{-1 \pm \boldsymbol{i}}{2}$$

# Some Generalizations:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 = 4ac$, there is *only one* root.

If $b^2 - 4ac$ is *negative*, the roots are ***complex***.