Vector Calculus

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Definition of Gradient operator: $\boxed{\vec{\boldsymbol{\nabla}} \equiv \hat{\boldsymbol{i}}\frac{\partial}{\partial x} + \hat{\boldsymbol{j}}\frac{\partial}{\partial y} + \hat{\boldsymbol{k}}\frac{\partial}{\partial z}}$

VECTOR IDENTITIES

TRIPLE PRODUCTS:

$$\begin{aligned}
\vec{\boldsymbol{A}} \cdot \left(\vec{\boldsymbol{B}} \times \vec{\boldsymbol{C}}\right) &= \vec{\boldsymbol{B}} \cdot \left(\vec{\boldsymbol{C}} \times \vec{\boldsymbol{A}}\right) = \vec{\boldsymbol{C}} \cdot \left(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}\right) \\
\vec{\boldsymbol{A}} \times \left(\vec{\boldsymbol{B}} \times \vec{\boldsymbol{C}}\right) &= \vec{\boldsymbol{B}}\left(\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{C}}\right) - \vec{\boldsymbol{C}}\left(\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}\right)
\end{aligned}$$

PRODUCT RULES:

$$\begin{aligned}
\vec{\nabla}(fg) &= f\left(\vec{\nabla} g\right) + g\left(\vec{\nabla} f\right) \\
\vec{\nabla}(\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}) &= \vec{\boldsymbol{A}} \times \left(\vec{\nabla} \times \vec{B}\right) + \vec{\boldsymbol{B}} \times \left(\vec{\nabla} \times \vec{A}\right) + \left(\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{\nabla}}\right)\vec{\boldsymbol{B}} + \left(\vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{\nabla}}\right)\vec{\boldsymbol{A}} \\
\vec{\boldsymbol{\nabla}} \cdot \left(f\vec{\boldsymbol{A}}\right) &= f\left(\vec{\nabla} \cdot \vec{A}\right) + \vec{\boldsymbol{A}} \cdot \left(\vec{\nabla} f\right) \\
\vec{\boldsymbol{\nabla}} \cdot (AB) &= \vec{\boldsymbol{B}} \cdot \left(\vec{\nabla} \times \vec{A}\right) - \vec{\boldsymbol{A}} \cdot \left(\vec{\nabla} \times \vec{B}\right) \\
\vec{\boldsymbol{\nabla}} \times \left(f\vec{\boldsymbol{A}}\right) &= f\left(\vec{\nabla} \times \vec{A}\right) - \vec{\boldsymbol{A}} \times \left(\vec{\nabla} f\right) \\
\vec{\boldsymbol{\nabla}} \times (AB) &= \left(\vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{\nabla}}\right)\vec{\boldsymbol{A}} - \left(\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{\nabla}}\right)\vec{\boldsymbol{B}} + \vec{\boldsymbol{A}}\left(\vec{\nabla} \cdot \vec{B}\right) - \vec{\boldsymbol{B}}\left(\vec{\nabla} \cdot \vec{A}\right)
\end{aligned}$$

SECOND DERIVATIVES:

$$\begin{aligned}
\vec{\boldsymbol{\nabla}} \cdot \left(\vec{\nabla} \times \vec{A}\right) &= 0 \\
\vec{\boldsymbol{\nabla}} \times \left(\vec{\nabla} f\right) &= 0 \\
\vec{\boldsymbol{\nabla}} \times \left(\vec{\nabla} \times \vec{A}\right) &= \vec{\boldsymbol{\nabla}}\left(\vec{\nabla} \cdot \vec{A}\right) - \nabla^2 \vec{\boldsymbol{A}}
\end{aligned}$$

FUNDAMENTAL THEOREMS

GRADIENT THEOREM:	$\int_a^b \left(\vec{\nabla} f\right) \cdot d\vec{\ell} = f(b) - f(a)$
DIVERGENCE THEOREM:	$\iiint \left(\vec{\nabla} \cdot \vec{A}\right) d\tau = \oiint \vec{A} \cdot d\vec{a}$
STOKES' (CURL) THEOREM:	$\left(\vec{\nabla} \times \vec{A}\right) \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{\ell}$

DELTA FUNCTION: $\vec{\boldsymbol{\nabla}} \cdot \left(\frac{\hat{\mathcal{R}}}{\mathcal{R}^2}\right) = -\nabla^2 \left(\frac{1}{\mathcal{R}}\right) = 4\pi \, \delta^3\left(\vec{\boldsymbol{\mathcal{R}}}\right)$