Solving Simple Equations

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In each case, explain which *principles* you use:

1. Solve for x: $x - 5 = 2$
ANSWER: add 5 to both sides of the equation to get $\boxed{x = 7}$.

2. Solve for x: $2x + 3 = 7$
ANSWER: first, subtract 3 from both sides of the equation to get $2x = 4$. Then divide both sides of the equation by 2 to get $\boxed{x = 2}$.

3. Solve for x: $x^2 - 5 = 4$
ANSWER: add 5 to both sides of the equation to get $x^2 = 9$. Then ask, "What number multiplied by itself gives 9?" Answer: $\boxed{x = \pm 3}$
(Note that -3 works just as well as $+3$.)

4. Solve for x: $x^2 = -1$
ANSWER: No real number multiplied by itself is -1, so we have to invent the *imaginary* number $i \equiv \sqrt{-1}$ giving $\boxed{x = \pm i}$.
(Note that $-i$ works just as well as $+i$.)

5. Solve for x: $x^2 - 2x = -1$
ANSWER: add 1 to both sides of the equation to get it into the standard form for a quadratic equation: $ax^2 + bx + c = 0$ with $a = 1$, $b = 2$ and $c = 1$. You may immediately recognize this as $(x-1)^2 = 0$, giving $x - 1 = 0$ or $\boxed{x = 1}$.
If not, you can always "plug in" to the Quadratic Formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Noting that $b^2 = 4ac = 0$ gives $\boxed{x = 2/2 = 1}$.
(This time there is only one result.)

6. Solve for x: $2x^2 - 3x - 4 = 0$
ANSWER: This time we go straight to the Quadratic Formula, with $a = 2$, $b = -3$ and $c = -4$:
$x = \dfrac{3 \pm \sqrt{9 + 32}}{4}$ or $\boxed{x = \dfrac{3}{4} \pm \dfrac{\sqrt{41}}{4}}$
(There is no need to try to simplify further.)