Derivatives of Functions

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1. Take the derivative with respect to x: $y(x) = 3$. ANSWER: The derivative (rate of change) of a *constant* is **zero**. (It's a *constant*. It *doesn't change*!) $\boxed{dy/dx = 0}$.

2. Take the derivative with respect to x: $y(x) = 2x + 1$. ANSWER: Use the *definition* of the derivative: $\frac{dy}{dx} \equiv \lim_{\Delta x \to 0} \frac{y(x+\Delta x) - y(x)}{\Delta x}$. In this case $y(x+\Delta x) = 2(x+\Delta x) + 1$ and $y(x) = 2x+1$, so $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{2x + 2\Delta x + 1 - (2x+1)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2\Delta x}{\Delta x}$ or $\boxed{\frac{dy}{dx} = 2}$.

(You could also just use the general formula $\frac{d}{dx} x^p = p x^{p-1}$.)

3. Take the derivative with respect to x: $y(x) = 3x^2$. ANSWER: Here you could also work out the algebra from the definition of a derivative, but it's easier to just use the PRODUCT LAW $\frac{d}{dx}[f(x) \times g(x)] = f'(x)g(x) + f(x)g'(x)$ with $f(x) = 3$ and $g(x) = x^2$, giving $f'(x) = 0$ and $g'(x) = 2x$ (from the power law formula above), so that $\boxed{\frac{dy}{dx} = 6x}$.

4. Take the derivative with respect to x: $y(x) = 2x^9$. ANSWER: The same approach as in the previous example gives $\boxed{\frac{dy}{dx} = 18x^8}$.

5. Take the derivative with respect to x: $y(x) = \frac{3}{x^3}$. ANSWER: Rewrite in negative-exponent notation: $y(x) = 3x^{-3}$ and use the same power-law formula as above: $\boxed{\frac{dy}{dx} = -9x^{-4}}$.

6. Take the derivative with respect to x: $y(x) = y_0\, e^{-\lambda x}$. ANSWER: Let $u(x) = -\lambda x$ $\left(\frac{du}{dx} = -\lambda\right)$ so that $e^{-\lambda x} = e^u$ and $\frac{d}{du} e^u = e^u$ (the *defining property* of the exponential function) and apply the CHAIN RULE: $\frac{d}{dx}\{y[u(x)]\} = \frac{dy}{du} \cdot \frac{du}{dx}$ to get $\boxed{\frac{dy}{dx} = -\lambda\, y_0\, e^{-\lambda x}}$.

7. Take the derivative with respect to x: $y(x) = e^{x^2}$. ANSWER: Again we define a "helper function" to use with the CHAIN RULE: in this case $u(x) = x^2$ so that $\frac{du}{dx} = 2x$ and again $y(x) = e^u$. Thus $\boxed{\frac{dy}{dx} = 2x\, e^{x^2}}$.

8. Take the derivative with respect to x: $y(x) = \ln x$

ANSWER: Remember that $\ln(x)$ is the *integral* of $1/x$, so $\boxed{\frac{dy}{dx} = \frac{1}{x}}$.