Differential Equations

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1. Which of the following **cannot** be expressed in terms of a DIFFERENTIAL EQUATION?

(a) Economic Inflation
(b) Radioactive Decay
(c) Propagation of Sound
(d) Propagation of Light
(e) Free Fall during Skydiving
(f) Infections during a Pandemic
(g) Population Growth

ANSWER: None of the above!

(a) Economic Inflation: $\dfrac{dV}{dt} = -kV$ where $V(t)$ is the value of a dollar as a function of time t and k is the inflation rate.

(b) Radioactive Decay: $\dfrac{dN}{dt} = -\lambda N$ where $N(t)$ is the number of the specified type of radioactive nuclei in a sample as a function of time t, $\lambda = 1/\tau$ and τ is the mean lifetime of such a nucleus.

(c) Propagation of Sound: $\dfrac{d^2P}{dx^2} - \dfrac{1}{c^2}\dfrac{d^2P}{dt^2}$ where $P(x,t)$ is the value of the air pressure as a function of position x and time t and c is the speed of sound.

(d) Propagation of Light: $\dfrac{d^2E}{dx^2} - \dfrac{1}{c^2}\dfrac{d^2E}{dt^2}$ where $E(x,t)$ is the electric field as a function of position x and time t and c is the speed of light.

(e) Free Fall during Skydiving: $\dfrac{dv}{dt} = g - \gamma v^2 - \lambda v$ where $v(t)$ is the downward velocity as a function of time t, $g = 9.81$ m/s^2 is the acceleration of gravity, γ is the turbulent drag coefficient and λ is the viscous drag coefficient.

(f) Infections during a Pandemic: $\dfrac{dN}{dt} = \dfrac{R_0}{T_{\rm incub}}N$ where $N(t)$ is the number of people infected as a function of time t, R_0 is the average number of new people infected by each victim and $T_{\rm incub}$ is the average incubation time.

(g) Population Growth: $\dfrac{dN}{dt} = \dfrac{n_b}{T_{\rm repr}}N - \dfrac{N}{T_{\rm life}}$ where $N(t)$ is the number of people as a function of time t, $T_{\rm life}$ is the average life expectancy, $T_{\rm repr}$ is the average time a woman is capable of reproduction, and n_b is the average number of babies each woman has in her lifetime.