# Fun with Functions

by *Jess H. Brewer*

October 26, 2020

In each case, explain which is the independent variable and which is the dependent variable, assign a mathematical symbol for each, explain what units they are measured in, and propose a mathematical function describing exactly how the dependent variable *depends* on the independent variable.

For instance, a car travelling at a constant velocity $v$ (in kph) goes a distance $x$ (in km) in a time $t$ (in hours): $\boxed{x(t) = vt}$.

**1.** Neglecting air friction, the downward velocity of a ball dropped from the Leaning Tower of Pisa will speed up by $g = 9.81$ m/s every second.
ANSWER: Let $v$ be the downward velocity in meters per second (m/s) and $t$ be the elapsed time in seconds (s); then $\boxed{v(t) = gt}$.

**2.** The vertical distance travelled by the ball in the previous problem will increase as $g/2$ times the square of the elapsed time.
ANSWER: Let $y$ be the vertical distance fallen in meters (m) and again let $t$ be the elapsed time in seconds (s); then $\boxed{y(t) = \frac{1}{2}gt^2}$.

**3.** The value of a share in a certain airline stock was \$100 on March 1, 2020, and has dropped at a constant rate since then. It is now worth half what it was on that day.
ANSWER: Let $S$ be the value of the share in dollars (\$) and $t$ be the elapsed time in days; then $\boxed{S(t) = S_0/2^{t/T}}$ where $S_0 = \$100$ and $T$ is the number of days since March 1, 2020.

**4.** If inflation is constant at 5% per year, how does the buying power of \$1 vary with time?
ANSWER: Let $B$ be the buying power of a dollar compared with that at time $t = 0$, $B_0$; then $\boxed{B(t) = B_0 \exp(-\lambda t)}$ where $t$ is in years and $\lambda = 0.05$ y$^{-1}$.

**5.** Helium slowly leaks right through the rubber membrane of a spherical helium balloon, causing its *volume* to decrease by a factor of two every day. What function describes the balloon's *radius* as a function of the number $t$ of days since it was filled if its initial diameter was 0.4 m?
ANSWER: First note that the volume $V$ is proportional to the *cube* of the radius $r$, $V = \frac{4}{3}\pi r^3$, so if the volume $V$ decreases by a factor of two every day — which we can write $V(t) = V_0/2^t$ where $V_0$ is the initial volume and $t$ is the number of elapsed days — then the radius $r = \left[\frac{3V}{4\pi}\right]^{1/3}$ varies as $\boxed{r(t) = \frac{r_0}{2^{t/3}}}$ where $r_0 = 0.2$ m.