Solving Integrals

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1. Find the integral with respect to x of $y(x) = 2x - 5$. ANSWER: We're looking for the function $f(x)$ whose *derivative* is $f'(x) = 2x - 5$. Now, within an arbitrary constant, the integral of a sum of terms is the sum of the integrals of those terms: $\int [a(x) + b(x)]\, dx = \int a(x)dx + \int b(x)dx$. In this case $a(x) = 2x$ and $b(x) = -5$. A constant like -5 is the derivative of that constant times x, and x itself is the derivative of $\frac{1}{2}x^2$, so $\boxed{\int [2x - 5]\, dx = x^2 - 5x + \text{const.}}$.

2. Find the integral with respect to x of $y(x) = x^3 + x^2$. ANSWER: $\frac{d}{dx}x^4 = 4x^3$ and $\frac{d}{dx}x^3 = 3x^2$, so $\boxed{\int \left[x^3 + x^2\right] dx = \frac{1}{4}x^4 + \frac{1}{3}x^3 + \text{const.}}$.

3. Find the integral with respect to x of $y(x) = -3/x$. ANSWER: The integral of a constant times a function of x is the constant time the integral of the function. (In simpler language, "You can just pull constants outside the integral!") $\int (-3/x)dx = -3 \int (1/x)dx$ and we know that $\ln(x)$ is the integral of $1/x$, so $\boxed{\int (-3/x)dx = -3\ln(x) + \text{const.}}$

4. Find the integral with respect to x of $y(x) = e^x$. ANSWER: The exponential function is *defined* by its property of being its own derivative — and therefore also its own integral: $\boxed{\int e^x dx = e^x + \text{const.}}$.

5. Find the integral with respect to x of $y(x) = e^{ix} = \cos(x) + i\sin(x)$. ANSWER: $\frac{d}{dx}e^{ix} = ie^{ix}$, so $\boxed{\int e^{ix} dx = \frac{e^{ix}}{i} + \text{const.}}$.[1]

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[1] Now, since $1/i = -i$ and $e^{ix} = \cos(x) + i\sin(x)$, we can also write $\boxed{\int e^{ix} dx = -i\cos(x) + \sin(x) + \text{const.}}$. By the same token, note that $\frac{d}{dx}[\cos(x) + i\sin(x)] = i\cos(x) - \sin(x)$, and since the real and imaginary parts can be differentiated independently, we have also found the derivatives of the trigonometric functions $\boxed{\frac{d}{dx}\cos(x) = -\sin(x)}$ and $\boxed{\frac{d}{dx}\sin(x) = \cos(x)}$. These are quite useful to know!