THE UNIVERSITY OF BRITISH COLUMBIA

Physics 108 Assignment # 1: THERMAL PHYSICS

Wed. 05 Jan. 2005 — finish by Wed. 12 Jan.

1. "STAT - EC": Consider the following simplified model of a sort of *stock market*: A given stock $\mathcal{S}$ has a total of N shares on the market for a fixed price ε. At a given time, n of these shares are bought and the remaining $N - n$ are unwanted. Thus the net investment in $\mathcal{S}$ is $U = n\varepsilon$. [Here ε and U are measured in *monetary* units, say dollars; I have used the same notation as for *energy* for reasons that will soon become evident.] To keep things simple, we shall assume that the *price* ε of a given stock *does not change*. Further, let's make the outrageous assumption that the stock market as a whole is *a priori equally likely to be found in any one of the fully specified states accessible to it* — *i.e.* that a given amount of capital is equally likely to be distributed amongst the various stocks in any of the possible ways that give the same total.[1]

(*a*) Invent a general definition for an economic analogue of *temperature* τ [measured in monetary units] that has the desired predictive power: that (given our starting assumptions) capital will tend to flow spontaneously from stocks with *higher* τ into others with *lower* τ and will stop flowing between two stocks only when they are in "economic equilibrium" — *i.e.* when they have the same "economic temperature" τ.

(*b*) Now assume that the entire market is in "economic equilibrium" and is so much larger than any of its parts that we may treat it as a "capital reservoir" $\mathcal{R}$ at an "economic temperature" of $\tau = \$100$. Consider one share of one stock, valued at $\varepsilon_1 = \$200$: What is the probability that it will be bought at any given time?

(*c*) Assuming that $\mathcal{R}$ is also huge compared to the entire offering of $N_1 = 1000$ shares of stock $\mathcal{S}_1$ valued at $\varepsilon_1 = \$200$, what is the expected total investment U_1 in $\mathcal{S}_1$ when $\tau = \$100$?

(*d*) If the economic temperature drops to $\tau = \$50$, which stock will be likely to have the most capital U invested in it, $\mathcal{S}_1$ with $N_1 = 1000$ shares at $\varepsilon_1 = \$200$ per share or $\mathcal{S}_2$ with $N_2 = 1000$ shares at $\varepsilon_2 = \$100$ per share?

2. MARS-EQUIVALENT ATMOSPHERIC PRESSURE: The composition of Mars' atmosphere is nominally 95.3% CO_2, 2.7% N_2, 1.6% Ar, 0.15% O_2 and 0.03% H_2O. Mean atmospheric pressure at the surface of Mars is 1-9 millibar, depending on altitude; the average is about 7 mb, compared to 1000 mb at sea level on Earth. *At what altitude* here on Earth would the atmospheric pressure be the same as that at the surface of Mars? (Assume an *isothermal* Earth atmosphere at 300 K. Are any other assumptions needed?)

3. ORTHO- *vs.* **PARA-HYDROGEN:** Molecular hydrogen, H_2, consisting of two protons bound together with two electrons, can form in either the "singlet" state called *parahydrogen*, in which the total spin (intrinsic angular momentum) of the molecule is zero, or in any one of three "triplet" states of *orthohydrogen*, in which the proton spins combine to make a total spin of $1\hbar$ (the fundamental unit of angular momentum). For this problem, all you need to know is that the three triplet states are *degenerate* — *i.e.* they all have the same *energy* relative to the singlet state, namely $\varepsilon_3 = 2.375 \times 10^{-21}$ J. (The energy ε_1 of the singlet state can be taken to be zero, for reference.) Assume that the spin degrees of freedom of the H_2 molecules are unaffected by, but are in thermal equilibrium with, all their other degrees of freedom (like translational, rotational or vibrational). In this case, what *fraction* f_3 of H_2 molecules will be found (on average) in *ortho* states

(*a*) at room temperature (300 K)?

(*b*) at the boiling point of liquid nitrogen at atmospheric pressure (77 K)?

(*c*) at the freezing point of molecular hydrogen at atmospheric pressure (14 K)?

[1] This is not consistent with current economic theory, which focusses on "rational agents." Here we assume totally mindless, random investment decisions.