The University of British Columbia

Physics 108 Assignment # 4:

GAUSS' LAW

Wed. 26 Jan. 2005 — finish by Wed. 2 Feb.

1. **FIELD WITHIN A UNIFORM CHARGE DISTRIBUTION:** The textbook shows how to use Gauss' Law to derive the radial (r) dependence of the electric field $E(r > R)$ *outside* charge distributions of spherical, cylindrical or planar symmetry, where R is the distance the charge distribution extends from the centre of symmetry — the radius of a charged *sphere* or *cylinder*, or half the thickness of an infinite *slab* of charge, respectively. Use similar arguments to show that, for each of these cases (a sphere, cylinder or a slab of uniform charge density), the electric field $E(r < R)$ *inside* the charge distribution is given in terms of the field $E(R)$ *at the boundary* of the charge distribution by

$$E(r < R) = \left(\frac{r}{R}\right) E(R).$$

2. **ATOMS AS SPHERES OF CHARGE:** In Rutherford's work on α particle scattering from atomic nuclei, he regarded the atom as having a point-like positive charge of $+Ze$ at its centre, surrounded by a spherical volume of radius R filled with a uniform charge density that makes up a total charge $-Ze$, making the atom as a whole electrically neutral. In this simple model, calculate the electric field strength E and the electric potential ϕ as functions of radius r and various constants. *Plot* your results for $0 < r \leq 2R$. (Choose $\phi \xrightarrow[r\to\infty]{} 0$.)

3. **TUBEWORLD:**[1] In a future interstellar voyage, you come across a gigantic hollow cylinder a million km long with an outer radius of 1000 km. It is spinning on its axis at an angular frequency ω, but with deft piloting you are able to land your spaceship at what looks like a dock halfway down the cylinder. As you approach, you find that the landing is easy because there is just enough gravitational attraction toward the surface to provide the acceleration you need to be in a circular orbit when you land at the dock. When you leave your craft in a spacesuit, you seem to be weightless. Now you see a door in the cylinder with a button labelled, "Press to open," so you do. A hatch opens and you step through into an airlock; the outer hatch closes, it fills with breathable air at 1 Earth atmosphere and then the inner hatch opens, revealing a hollow interior filled with air and light from a long line source down the axis of the cylinder. As you stand on the inner surface you experience an apparent Earth-normal gravity of "1 g" pulling you toward the surface (away from the axis). Many interesting creatures live here, but before we go meet them I have a few Answers and Questions. First the Answers: Virtually all the mass of the cylinder is in the thin shell through which you just stepped. It is made of a *very* dense material! Although this all requires technology beyond our present grasp,[2] none of it is in conflict with the "known laws of physics" and you have all the tools you need to answer the Questions:

   (a) How long does it take for the cylinder to spin once about its axis?

   (b) What is the net mass of the cylinder, and how does it compare with the mass of the Earth? With the mass of the Sun?

[1] The *Ringworld* novels of Larry Niven inspired this adaptation including gravity and a more enclosed geometry.

[2] [Arthur C.]Clarke's Law: "Any sufficiently advanced technology is indistinguishable from magic."