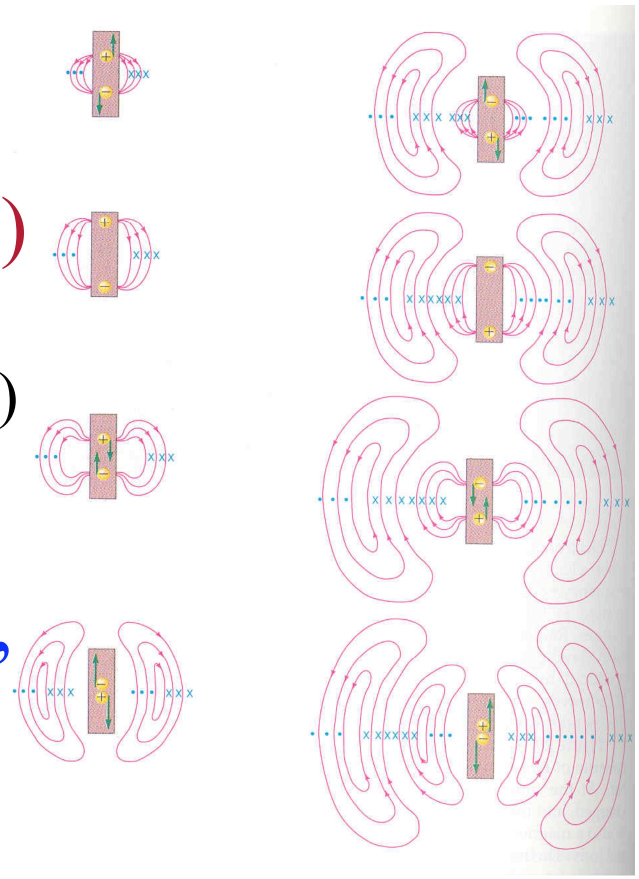
Retarded Potentials

•Retarded Potentials

•Jefimenko's Eqns (retarded fields)

(Oscillating charges generate EM radiation)



Presentation today:

Ricky Chu "Aurora Borealis"

(I must stop at 9:50 - 0:14 -.01= 9:35)

Retarded Potentials

•When we hear a jet overhead, the noise comes from not where we see the jet, but from where the jet was when it emitted the sounds waves that reach us now.

•When we look deep into the night sky, we see stars as they were billions of years ago when they emitted the light that reaches our eyes today (and not as they are NOW)

•Information, be it sounds waves or light, travels at a finite speed: we can only detect changes in a distribution of source charges and currents some after these waves leave their source.

Retarded Time

The electric or magnetic field at a given time and place depends on the distribution of charges and currents at some time in the past.

There is a time delay for information to reach us here and now. The "retarded time" is the time it takes for "electromagnetic news" (as Griffiths calls it) to travel from the charge to the field point of interest (at speed c).

$$t_r \equiv t - \frac{\mathscr{r}}{c}$$

EM potentials:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,')\, d\tau'}{\mathscr{r}}$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\, d\tau'$$

become retarded potentials:

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,',t_r)\, d\tau'}{\mathscr{r}}$$

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,',t_r)}{\mathscr{r}}\, d\tau'$$

Retarded Time

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,',t_r)\,d\tau'}{\mathscr{r}}$$

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,',t_r)}{\mathscr{r}}\,d\tau'$$

$$t_r \equiv t - \frac{\mathscr{r}}{c}$$

We must show that the retarded potentials satisfy:

- Lorentz invariance
- the Maxwell's equations (or equivalent eqns for potentials)

before we can claim they are correct

EM fields:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')\,d\tau'}{\mathscr{r}^2}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau'$$

$$\vec{E}(\vec{r},t) \neq \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,',t_r)\,d\tau'}{\mathscr{r}^2}$$

No!!

$$\vec{B}(\vec{r},t) \neq \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,',t_r)\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau'$$

No!!

Retarded Potentials

- Maxwell's equations in Lorentz Gauge

$$-\nabla^2 \vec{A} + \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = \mu_0 \vec{J}$$

$$-\nabla^2 V + \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = \frac{\rho}{\epsilon_0}$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,',t_r)\,d\tau'}{\mathscr{r}}$$

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,',t_r)}{\mathscr{r}}\,d\tau'$$

Potentials depend on $\vec{\mathscr{r}}$ directly, AND on Retarded time, which has indirect $\vec{\mathscr{r}}$ dependance

$$\vec{\nabla} V = \vec{\nabla}\left[\frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,',t_r)}{\mathscr{r}}\,d\tau'\right] = \frac{1}{4\pi\epsilon_0}\int \vec{\nabla}\,\frac{\rho(\vec{r}\,',t_r)}{\mathscr{r}}\,d\tau'$$

$$\vec{\nabla} V = \frac{1}{4\pi\epsilon_0}\int \left[\frac{1}{\mathscr{r}}\vec{\nabla}\rho(\vec{r}\,',t_r) + \rho\,\vec{\nabla}\left(\frac{1}{\mathscr{r}}\right)\right] d\tau'$$

$$\dot{\rho} = \frac{\partial \rho}{\partial t_r}$$

$$\vec{\nabla}\rho(\vec{r}\,',t_r) = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\rho(\vec{r}\,',t_r)$$

Retarded Potentials

- Maxwell's equations in Lorentz Gauge

$$-\nabla^2\vec{A}+\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}=\mu_0\vec{J}\qquad\qquad \vec{A}(\vec{r},t)=\frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}\,',t_r)\,d\tau'}{\mathscr{r}}$$

$$-\nabla^2 V+\frac{1}{c^2}\frac{\partial^2 V}{\partial t^2}=\frac{\rho}{\epsilon_0}\qquad\qquad V(\vec{r},t)=\frac{1}{4\pi\epsilon_0}\int\frac{\rho(\vec{r}\,',t_r)}{\mathscr{r}}\,d\tau'$$

Continue, do chain rule:

$$\vec{\nabla}\rho(\vec{r},t_r)=\hat{x}\frac{\partial\rho}{\partial t_r}\frac{\partial t_r}{\partial x}+\hat{y}\frac{\partial\rho}{\partial t_r}\frac{\partial t_r}{\partial y}+\hat{z}\frac{\partial\rho}{\partial t_r}\frac{\partial t_r}{\partial z}\qquad\qquad \dot{\rho}=\frac{\partial\rho}{\partial t_r}$$

$$\vec{\nabla}\rho(\vec{r},t_r)=\hat{x}\dot{\rho}\frac{\partial t_r}{\partial x}+\hat{y}\dot{\rho}\frac{\partial t_r}{\partial y}+\hat{z}\dot{\rho}\frac{\partial t_r}{\partial z}$$

$$=\dot{\rho}\left[\hat{x}\frac{\partial t_r}{\partial x}+\hat{y}\frac{\partial t_r}{\partial y}+\hat{z}\frac{\partial t_r}{\partial z}\right]$$

Recognize gradient of t_r:

$$=\frac{\partial\rho}{\partial t_r}\vec{\nabla}t_r=\frac{\partial\rho}{\partial t_r}\vec{\nabla}\left(t-\frac{\mathscr{r}}{c}\right)$$

Retarded Potentials

- Continue: $$\vec{\nabla} V = \frac{1}{4\pi\epsilon_0}\int\left[\frac{1}{\mathscr{r}}\vec{\nabla}\rho(\vec{r}',t_r) + \rho\vec{\nabla}\frac{1}{\mathscr{r}}\right]d\tau'$$

$$\vec{\nabla}\rho = -\frac{\dot{\rho}}{c}\hat{\mathscr{r}}$$

$$\vec{\nabla}\frac{1}{\mathscr{r}} = -\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\vec{\nabla} V = \frac{1}{4\pi\epsilon_0}\int\left[\frac{1}{\mathscr{r}}\left(-\frac{\dot{\rho}}{c}\hat{\mathscr{r}}\right) + \rho\left(-\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\right]d\tau'$$

$$= \frac{-1}{4\pi\epsilon_0}\int\left[\frac{\dot{\rho}}{c}\frac{\hat{\mathscr{r}}}{\mathscr{r}} + \rho\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right]d\tau'$$

Now $\nabla^2 V$:

$$\nabla^2 V = \vec{\nabla}\cdot\left\{\frac{-1}{4\pi\epsilon_0}\int\left[\frac{\dot{\rho}}{c}\frac{\hat{\mathscr{r}}}{\mathscr{r}} + \rho\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right]d\tau'\right\}$$

$$\nabla^2 V = \frac{-1}{4\pi\epsilon_0}\int\left[\vec{\nabla}\cdot\left(\frac{\dot{\rho}}{c}\frac{\hat{\mathscr{r}}}{\mathscr{r}}\right) + \vec{\nabla}\cdot\left(\rho\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\right]d\tau'$$

$$\nabla^2 V = \frac{-1}{4\pi\epsilon_0}\int\left[\frac{\dot{\rho}}{c}\left(\vec{\nabla}\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}}\right) + \frac{\hat{\mathscr{r}}}{\mathscr{r}}\cdot\left(\vec{\nabla}\frac{\dot{\rho}}{c}\right) + \rho\left(\vec{\nabla}\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) + \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\cdot\vec{\nabla}\rho\right]d\tau'$$

Retarded Potentials

Bits and pieces:

$$\vec{\nabla}\rho = -\frac{\dot{\rho}}{c}\hat{\mathscr{r}} \qquad \vec{\nabla}\frac{\dot{\rho}}{c} = \frac{1}{c}\vec{\nabla}\dot{\rho} = -\frac{\ddot{\rho}}{c^2}\hat{\mathscr{r}} \qquad \vec{\nabla}\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}} = \frac{1}{\mathscr{r}^2} \qquad \vec{\nabla}\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = 4\pi\delta^3(\vec{\mathscr{r}})$$

So finally:

$$\nabla^2 V = \frac{-1}{4\pi\varepsilon_0}\int\left\{\frac{\dot{\rho}}{c}\frac{1}{\mathscr{r}^2} + \frac{\hat{\mathscr{r}}}{\mathscr{r}}\cdot\left[-\frac{\ddot{\rho}}{c^2}\hat{\mathscr{r}}\right] + 4\pi\rho\delta^3(\vec{\mathscr{r}}) + \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\cdot\left[-\frac{\dot{\rho}}{c}\hat{\mathscr{r}}\right]\right\}d\tau'$$

Perform dot products:

$$\nabla^2 V = \frac{-1}{4\pi\varepsilon_0}\int\left\{\frac{\dot{\rho}}{c\mathscr{r}^2} - \frac{\ddot{\rho}}{c^2\mathscr{r}} + 4\pi\rho\delta^3(\vec{\mathscr{r}}) - \frac{\dot{\rho}}{c\mathscr{r}^2}\right\}d\tau'$$

$$\frac{1}{4\pi\varepsilon_0}\int\frac{\ddot{\rho}}{c^2\mathscr{r}}d\tau' = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left[\frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\vec{r}',t_r)}{\mathscr{r}}d\tau'\right] = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}V(\vec{r},t)$$

Retarded Potentials

Finally:

$$\int \rho(\vec{r}\,', t_r)\,\delta^3(\vec{\mathscr{r}})\,d\tau' = \rho(\vec{r}, t)$$

$$\nabla^2 V = \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} - \frac{\rho(\vec{r}, t)}{\varepsilon_0}$$

Yay! The retarded potential satisfies Maxwell’s eqns in the Lorentz gauge. (You can show the vector A field does too - same arguments - only 3 times, once for each vector component)

Jefimenko's Equations

Yay! The retarded potential satisfies Maxwell's eqns in the Lorentz gauge. (You can show the vector A field does too - same arguments)

$$\frac{\partial \vec{A}}{\partial t} = \frac{\partial}{\partial t}\int \frac{\vec{J}(\vec{r}',t_r)}{\mathscr{r}}\,d\tau' = \int \frac{\dot{\vec{J}}(\vec{r}',t_r)}{\mathscr{r}}\,d\tau' =$$

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\left[\left(\frac{\rho(\vec{r}',t_r)}{\mathscr{r}^2} + \frac{\dot{\rho}(\vec{r}',t_r)}{c\mathscr{r}}\right)\hat{\mathscr{r}} - \frac{\dot{\vec{J}}(\vec{r}',t_r)}{c^2\mathscr{r}}\right]d\tau'$$

$$\vec{B} = \vec{\nabla}\times\vec{A} = \frac{\mu_0}{4\pi}\int\left[\frac{1}{\mathscr{r}}\left(\vec{\nabla}\times\vec{J}\right) - \vec{J}\times\left(\vec{\nabla}\frac{1}{\mathscr{r}}\right)\right]d\tau'$$

$$\vec{B} = \vec{\nabla}\times\vec{A} = \frac{\mu_0}{4\pi}\int\left[\frac{1}{\mathscr{r}}\left(\dot{\vec{J}}(\vec{r}',t_r)\times\frac{\hat{\mathscr{r}}}{c}\right) + \vec{J}(\vec{r}',t_r)\times\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\right]d\tau'$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int\left[\frac{\dot{\vec{J}}(\vec{r}',t_r)}{c\mathscr{r}}\times\hat{\mathscr{r}} + \frac{\vec{J}(\vec{r}',t_r)}{\mathscr{r}^2}\times\hat{\mathscr{r}}\right]d\tau'$$

Presentations in PHYS 401

Today's presentation:

Ricky Chu "Aurora Borealis and Australis"

Upcoming :

Mar. 11 Tudor Costin Relativistic Potentials or Abraham-Lorentz Radiation Reaction

If you need me to print some slides for you (PDF files please!), or get a demo, or if you want to use the LCD projector, please let me know at about 24 hours beforehand.

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