

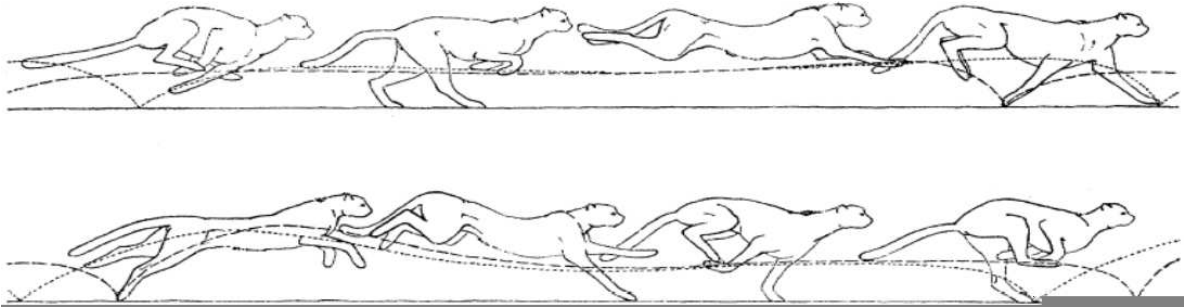
Physics 438 Assignment # 4:
MORE MECHANICS & METABOLISM
SOLUTIONS:

Thu. 15 Feb. 2007 — finish by Thu. 1 Mar.

Please hand in one assignment per group and list the names & Email addresses of all group members at the top of each sheet. During Reading Break you can gather information from the literature and attempt each problem on your own; then when you return you can pool your information and ideas with the rest of your team and your final solutions should emerge quickly. As always, *if you think some necessary information is missing, make a reasonable assumption*. But always write down what that assumption is. Remember to estimate your *uncertainty* in any measured quantity, and don't forget to specify all *units*. If possible, justify your input. For original comments you may score bonus points!

1. THE ENERGY LOSS OF A HUNT:

- (a) Search the literature to collect data about a fast hunter such as wild dog, leopard, or lion.¹ Find mass M , top speed U_t , length of leg L , length of step while foot is on the ground S , period $T = 2\pi/\omega$ (either from $T = 1/n$ where n is the number of steps per second, or from the distance λ between two imprints of the feet on the ground: $T = \lambda/U_t$, duration τ of the hunt, or the range $R = U_t\tau$. Treat the motion of the rear leg like simple harmonic motion where the position of the foot relative to the vertical is $x = A \sin(\omega t)$, where $A = S/2$ is the amplitude, and $U = \omega A \cos(\omega t)$ is the velocity of the foot. The maximum foot velocity is $U_{\max} = \omega A$, the instantaneous acceleration of the foot is $a = \omega^2 A \sin(\omega t)$, and the maximum foot acceleration is $a_{\max} = \omega^2 A$. **ANSWER:**



Cheetah, *Acinonyx jubatus*. (Fig. from Hildebrand, 1962)²

Body mass: $M = 40$ kg (Marker and Dickman, 2003).³
 Average top speed: $U_t = 14.8$ m/s (Hildebrand, 1962).
 Limb Length: $L = 0.79$ m (Marker and Dickman, 2003).
 Average period: $T = 0.31$ s (Hildebrand, 1962).
 Angular deflection of the limb: $\phi = 30^\circ$ (Ahlborn, 2004).
 Distance covered during hunt: $X_{\text{hunt}} = 60$ m.
 Duration of hunt: $\tau = 5$ s.

The parameters that control the speed U are stride length λ and stride frequency f (Alexander, 2003):⁴
 $U = \lambda \times f$. Therefore, we can calculate a stride length for a given speed and stride frequency:
 $f = 1/T = 1/0.31$ s = 3.3 Hz and (at top speed) $\lambda = U_t/f = (14.8$ m/s)/(3.3 Hz) or $\lambda = 4.48$ m.

¹Note: some misprints in the assignment as handed out have been corrected in this version.

²M. Hildebrand, 1962: "Further studies on locomotion of the cheetah", *Journal of Mammalogy* **42**, 84-91 (1962).

³L.L. Marker and A.J. Dickman, 2003: "Morphology, physical condition and growth of the cheetah (*Acinonyx jubatus jubatus*)", *Journal of Mammalogy* **84**, 840-850 (2003).

⁴R.McN. Alexander, 2003: *Principles of Animal Locomotion* (Princeton University Press, 2003).

To get the length S of a “step” requires some knowledge of the leg motion. If we treat this as the swing of a straight rod of length L through an arc subtending $2\phi = 60^\circ$ at constant ω (ignoring the fact that this would be a pretty bumpy ride), we can set $U = L\omega$ and pretend that the leg just keeps swinging around in a circle at the same ω (like in a “Roadrunner” cartoon), in which case the step length S is the same fraction of the total stride length λ as 60° is of 360° — namely $1/6$. This immediately gives (at top speed) a step length $S = \lambda/6$ or

$S \approx 0.75 \text{ m}$. Alternatively, we can ignore the details of how the leg might bend during the step and simply assume that it begins with the leg stretched out straight at a 30° angle in front (so that the shoulder is $L \sin 30^\circ = 0.79/2 \text{ m}$ behind the foot at the moment the foot first touches the ground) and ends with the leg stretched out straight at the same angle behind the shoulder (so that the shoulder is $0.79/2 \text{ m}$ ahead of the foot as it leaves the ground) for a total step length of $S = 0.79 \text{ m}$, consistent with the estimate above.

- (b) Assuming that the maximum acceleration of the foot is equal to the maximum acceleration of the body, find the acceleration time t_1 needed to reach top speed $U_t = at_1$. **ANSWER:** The maximum acceleration of the foot (in the context of simple harmonic motion) is $a_{\max} = A\omega^2$ where A is the amplitude, $A = L \sin \phi = (0.79 \text{ m})(\sin 30^\circ) \Rightarrow a_{\max} = A\omega^2 = (0.79 \text{ m})(3.3 \text{ Hz})^2 = 8.6 \text{ m/s}^2$. Of course, this maximum horizontal acceleration occurs at the end of the range of motion (*e.g.*, for the “Road Runner” picture, when the leg is horizontal) and at mid-stride the horizontal acceleration in this model would be zero; so we are bound to be overestimating the maximum acceleration by something like a factor of two. The problem tells us to assume that the acceleration of the foot is equal to the acceleration of the body, $a_{\max} = a$, so the time to reach top speed is $t_1 = U_t/a = (14.8 \text{ m/s})/(8.6 \text{ m/s}^2)$ or $t_1 = 1.7 \text{ s}$.⁵
- (c) Determine the speed-up distance s_1 (similar to the free fall distance $s = \frac{1}{2}gt^2$). **ANSWER:** The speed-up distance is similar to free fall distance, but instead of gravity we use our calculated acceleration: $s_1 = \frac{1}{2}at_1^2 = (0.5)(8.6 \text{ m/s}^2)(1.7 \text{ s})^2$ or $s_1 = 7.3 \text{ m}$.
- (d) Calculate the kinetic energy K which the animal attains at top speed. **ANSWER:** This is straightforward: $K = \frac{1}{2}MU_t^2 = (0.5)(40 \text{ kg})(14.8 \text{ m/s})^2$ or $K = 4380 \text{ J}$.⁶
- (e) Assuming that K was obtained by the application of an average force F acting during the time t_1 or over the distance s_1 , the energy equation reads $K = F \times s_1$, from which F can be found. **ANSWER:** Given $K = 4380 \text{ J}$ and $s_1 = 7.3 \text{ m}$, we use $F = K/s_1$ to get $F = 600 \text{ N}$.
- (f) Find the total energy expense of the hunt, $E_{\text{tot}} = F \times X_{\text{hunt}}$ and express it in mass of body fat burned, taking into account a reasonable inefficiency of converting the body fat into muscle fuel.⁷ **ANSWER:** $E_{\text{tot}} = FX_{\text{hunt}} = (600 \text{ N})(60 \text{ m})$ or $E_{\text{tot}} = 36 \text{ kJ}$. Such a short sprint would not normally be reflected in the conversion of fat to fuel, but rather governed by anaerobic metabolic pathways. The muscle biochemistry and histology of cheetah locomotor muscles show a high capacity for glycolysis and indicate that they are well equipped for anaerobic metabolism.⁸ Nevertheless, the energy has to eventually come from some form of “chemical fuel”, and if the cheetah had no other source than its own body fat, it would have to “burn” a mass $\Delta M = E_{\text{tot}}/\eta h$ where $\eta \approx 25\%$ is the efficiency of utilization for work and $h = 32 \times 10^6 \text{ J/kg}$ is the specific energy content of fat. This gives $\Delta M = 0.45 \times 10^{-2} \text{ kg}$ or $\Delta M = 4.5 \text{ g}$. (The cheetah needs to be successful at least once in every few hundred tries or she will get pretty skinny!)

⁵This seems pretty quick, but within reason. A Thomson’s gazelle reaches 15 m/s starting from rest in approximately 5 seconds (Elliott *et al.*, 1977); but the gazelle is more of a middle-distance runner, whereas the cheetah is Nature’s ultimate “power sprinter”. Assuming that the acceleration of the body and the foot are equal may be an oversimplification. If we throw in a factor of two to give the average acceleration between 8.6 m/s^2 and zero, we get $t_1 = 3.4 \text{ s}$, which is still a little quicker than the gazelle but a lot more challenging for the cheetah.

⁶This value is consistent with $K_h = 270 \text{ J}$ calculated for *just the hindlimb* of a cheetah by M. Hildebrand and J.P. Hurley, 1985: “Energy of the oscillating legs of fast-moving cheetah, pronghorn, jackrabbit and elephant”, *Journal of Morphology* **184** 23-31 (1985).

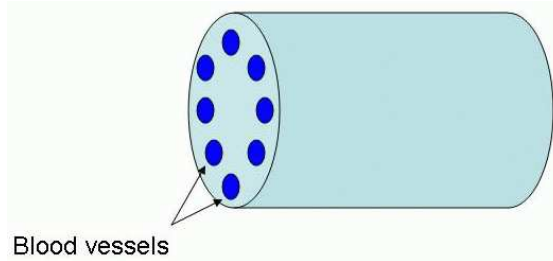
⁷There is an assumption in the wording of the question that the same force F is applied throughout, even after the cheetah has reached top speed U_t and ceased accelerating. On one hand, as any sprinter can attest, running at top speed is not effortless, even though acceleration is finished; there is friction to overcome, and merely moving one’s legs back and forth that fast takes an enormous amount of work! On the other hand, this does not translate into an externally applied net force F . However, if we approximate the net rate of work as being the same (namely the maximum the animal can manage) throughout the run, and we know that it mostly goes into acceleration at the beginning, we can get a fair estimate of the net energy expended by pretending that the original F is exerted through the entire distance X_{hunt} .

⁸T.M. Williams, G.P. Dobson, O. Mathieu-Costello, D. Morsbach, M.B. Worley and J.A. Phillips, 1997: “Skeletal muscle histology and biochemistry of an elite sprinter, the African cheetah”, *Journal of Comparative Physiology B* **167**, 527-535 (1987).

2. **LANDING ON YOUR FEET:** Two young people jump down from a height of $h = 2.0$ m onto the forest floor which has a mud hole and a rocky flat. The boy ($M_b = 70$ kg) lands with stiff legs on the muddy ground, which “gives” so that he depresses the ground by $\Delta y = 5$ cm when being decelerated from the impact velocity U_0 to rest.

- (a) Determine the impact speed U_0 . **ANSWER:** You can use either the constant acceleration equation $U_0^2 = 2gh$ or the conservation of energy $\frac{1}{2}M_bU_0^2 = M_bgh$ to get $U_0 = 6.26$ m/s.
- (b) Calculate the deceleration (negative acceleration) of the the boy’s center of mass during landing, and determine the average force on the soles of his shoes during impact. **ANSWER:** We have little choice but to assume the mud exerts a constant force F_b during the boy’s impact, absorbing his entire kinetic energy $K = \frac{1}{2}M_bU_0^2$ as work $W = F_b\Delta y$ done on the mud. Thus $F_b = M_bU_0^2/2\Delta y = (70 \text{ kg})(6.26 \text{ m/s})^2/(2 \times 0.05 \text{ m})$ or $F_b = 27,468$ N or 40 times his weight. *Ouch!*
- (c) The girl ($M_g = 65$ kg) lands on the rock, but in order to reduce the impact force she lands with soft knees and moves her center of gravity relative to her feet by $\Delta y = 0.5$ m as she lands. What is the average impact force on the soles of her feet during the landing? **ANSWER:** The velocity on impact U_0 is the same for both. For the young woman we use the same equation, except that Δy is 10 times bigger and her mass M_g is less, so the average force is more than ten times smaller: $F_g = M_gU_0^2/2\Delta y = (65 \text{ kg})(6.26 \text{ m/s})^2/(2 \times 0.5 \text{ m})$ or $F_g = 2,551$ N or 4 times her weight.

3. **DIFFUSION AND METABOLIC RATES OF A NUDIBRANCH:** A certain nudibranch of mass $M = 0.005$ kg living in 10°C waters off Vancouver Island carries its gills (mass m) outside its body. Assume that (i) the gills are 10% of the total body mass, (ii) the gills are tree-like structures that have branches with an average diameter of $D = 500 \mu\text{m}$, filled with fine capillary vessels of diameter $d = 10 \mu\text{m}$ right under the skin of the gills. (See sketch.)



- (a) Determine the surface area A of the gills that contributes to the diffusion exchange of oxygen. **ANSWER:** Mass of nudibranch: $M = 0.005$ kg. Mass of gills: $m = 0.0005$ kg. Gill volume $V_g = (0.0005 \text{ kg})/(1000 \text{ kg/m}^3) = 5 \times 10^{-7} \text{ m}^3$. Gill ‘trunk’ diameter $d_g = 500 \times 10^{-6} \text{ m}$. Gill capillary diameter $d_c = 10 \times 10^{-6} \text{ m}$. If we treat the entire gill volume as one long cylinder of diameter d_g , its length L_g can be deduced from its volume $V_g = \pi(d_g/2)^2L_g$: $L_g = V_g/\pi(d_g/2)^2 = 2.55$ m. Then we can calculate its surface area as $A_g = \pi d_g L_g = \pi(500 \times 10^{-6} \text{ m})(2.55 \text{ m})$ or $A_g = 4 \times 10^{-3} \text{ m}^2$.
- (b) Calculate the diffusion flow rate of oxygen \dot{N}_{O_2} [molecules/sec] into the gills. (See Sect. 4.1.3; assume first that the blood returning from the body into the gills is completely deoxygenated.) **ANSWER:** The diffusion flux $\dot{N}_{O_2} = A_g D \Delta n / \Delta x$, Where A_g is the area over which diffusion occurs, D is the diffusion coefficient and Δn is the change in concentration over a given distance Δx . Here $\dot{N}_{O_2} = [(4 \times 10^{-3} \text{ m}^2)(18 \times 10^{-10} \text{ m}^2/\text{s})(1.63 \times 10^{23} \text{ molecules O}_2/\text{m}^3 \text{ H}_2\text{O})]/(10 \times 10^{-6} \text{ m})$ or $\dot{N}_{O_2} = 1.17 \times 10^{17} \text{ molecules O}_2/\text{s}$.
- (c) Use the relation between \dot{N} and Γ which is derived in Section 4.3.3 to determine the metabolic rate of the nudibranch. **ANSWER:** $\dot{N}_{O_2} = J_{O_2}/M_{O_2} = (3.4\Gamma)/(M_{O_2}\Delta H)$. Thus $1.17 \times 10^{17} \text{ molecules O}_2/\text{s} = (3.4\Gamma)/[(32 \times 1.6 \times 10^{-27} \text{ kg})(30 \text{ MJ/kg})]$ so $\Gamma = [(1.17 \times 10^{17} \text{ molecules O}_2/\text{s})(32 \times 1.6 \times 10^{-27} \text{ kg})(30 \text{ MJ/kg})]/3.4 = 5.3 \times 10^{-8} \text{ MJ/s}$ or $\Gamma = 0.053 \text{ W}$.
- (d) Assuming that for these animals $\Gamma = aM^{3/4}$, determine the metabolic constant a . **ANSWER:** $\Gamma = aM^{0.75} \Rightarrow a = \Gamma/M^{0.75} = 0.053/0.005^{0.75}$ or $a = 2.82$.

- (e) Look into the literature to assess if the assumption is reasonable that the blood returning from the body into the gills is completely deoxygenated. If not, how would this change your answers? **ANSWER:** Many references show curves similar to those in Fig. 4.17 on p. 145 of the textbook, indicating that essentially all h emoglobin oxygen bonding sites are empty when in equilibrium with a reservoir at zero partial pressure of O_2 . However, living tissue is unlikely to have zero partial O_2 pressure at any time, so this is not too helpful.

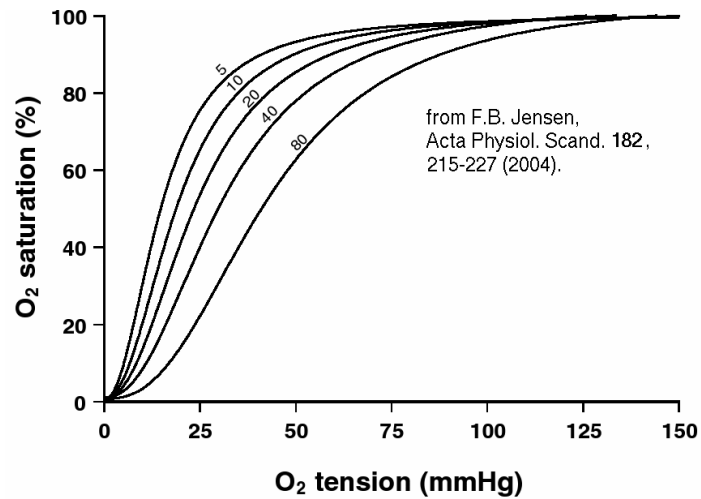


Figure 1 Influence of CO_2 on O_2 equilibrium curves of dog blood at $38\text{ }^\circ\text{C}$. Numbers at each curve show the CO_2 tension in mmHg. The equilibrium curves were drawn using the data listed in Table IV of the paper by Bohr *et al.* (1904).

The ‘‘Bohr effect’’ (see above) helps squeeze more O_2 out of h emoglobin sites by filling them with CO_2 supplied by the same tissues, which helps but does not really answer the question. Figure 4.18(b) on p. 146 suggests that the equilibrium O_2 partial pressure in venous blood returning to the lungs is about 5 kPa, compared with about 12.5 kPa for oxygenated blood leaving the lungs, which means that venous blood is still about 40% oxygenated. Since Δn is the O_2 concentration difference between the water (which we assume has the same O_2 partial pressure as air, namely about 20 kPa) and the deoxygenated blood, if the latter has $p_{O_2} \approx 5$ kPa instead of zero as assumed above, it will reduce \dot{N}_{O_2} by a factor of $15/20 = 0.75$, and the other calculations should be adjusted accordingly.