It Is Raining Cats

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Abstract

Cats are known to right themselves by rotating their bodies while falling through the air and despite being released from almost any position, they are known to have the ability to land on their feet. Several mechanisms have been proposed to explain how cats are able to reorient themselves in the air. All observations and measurements were made on my averaged sized house cat, Wong Wong. A cat was simplified into three two part models and the models were used to test whether or not a one joint, two segment model would be sufficient in explaining how a cat is able to rotate in the air to land feet first when dropped from an upside down position in 0.3 s. The first model consists of one segment forming the tail and another forming the body and the head and the results show that the cat would have to have an angular velocity of 331 rad/s and rotate 15 times. The second model has the cat divided into a head and body and the results show that the head would have to rotate at 1141 rad/s about 54.5 times. The third model divides the cat in half with the first half consisting of head and upper body and the second half consisting of the tail and the lower body and results show that the cat would have to rotate at 15 rad/s and it would have to rotate 257 degrees. The models were restricted to rigid two cylinder, coupled systems that revolved around one axis to allow the application of simple physics equations. The calculations suggest that the cat can rely solely on a two segment one joint system to rotate itself in the air using the third model but not the first two.

Introduction

A cat with no initial rotational momentum is somehow able to rotate itself in the air by contorting its body and it is widely known that they can do it. Part of this is due to their bodies being highly flexible. My house cat can curl up to measure about 30cm in diameter or stretch out to measure 1 meter in length. Many other animals do not have this ability. A cat’s flexibility combined with a righting sensory mechanism allows them to be able execute this instinctive righting maneuver but how does a cat mechanically do this? Given that a cat, like any living system, is very complex, is it possible for a two segment, one jointed system to model how a cat rights itself in the air?

Methods

I came up with 3 mechanisms to address the question. The calculations that all three models are based upon is rotational momentum or torque. The equation used for rotational momentum is:

$$\Sigma\tau = I\acute{\omega}$$

Specifically, any falling object that has no external force applied to it will display conservation of rotational momentum. Furthermore, I assume that a cat has no initial rotational momentum so that the net rotational momentum of the object at any point in its fall must also be zero.

$$\Sigma\tau = I_{(initial)}\acute{\omega}_{(initial)} = I_{(final)}\acute{\omega}_{(final)} = 0$$

Where I is the inertia in kilograms meters squared:

$$I \propto ml^2$$

Where $\acute{\omega}$ is the angular velocity in radians per second:

$$\acute{\omega} = \phi/t$$

The time it takes for a cat to right itself in the air is 0.3 seconds and this is determined experimentally as follows. A video was shot of my cat falling upside down out of my hands and from video replay it takes her 7 frames to turn over. Each frame has a frequency of about 30 Hz so the period is:

$$T = 1/f = 1/30 \text{ Hz} = .033\text{s}$$

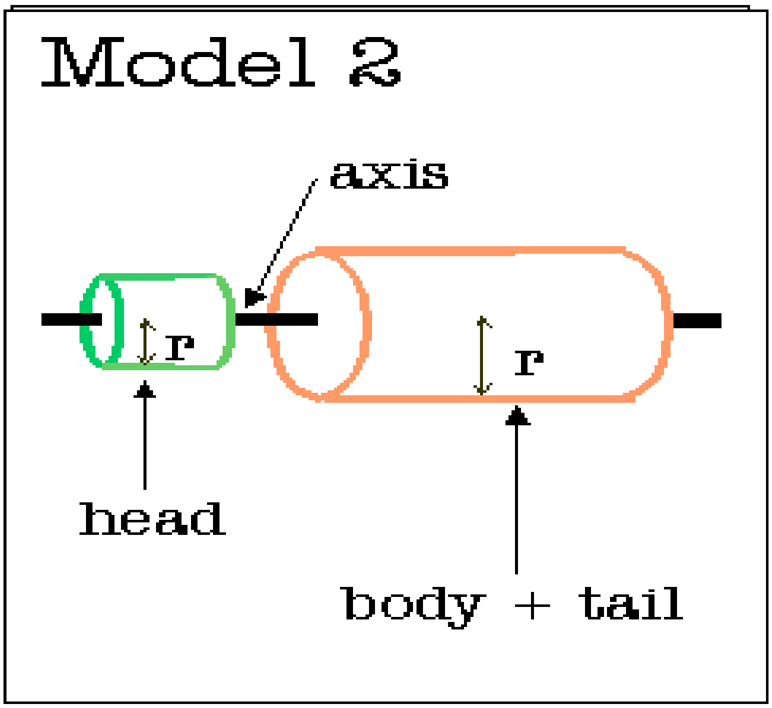
$$T(\# \text{ of frames}) = 2.3\text{s}$$

However, the cat in the video seems to be leaning off my hands as it is going into the fall and so she does have a small initial rotational momentum going into the fall which would make her turn in the air faster then she would without any rotational momentum. Thus the number used is larger to make up for this discrepancy.

The legs of the cat were assumed to be pulled in toward the body for all three models. The body/feet of the cat is set to turn 180° for all models in other words the cat does rotate continuously until it hits the ground, instead it turns 180° and stops. Where possible, direct measurements were made on Wong Wong. She was placed on a regular household scale and found to weigh 4.54 Kg. With a ruler, I measured the height of her shoulders from the floor when she was sitting on the floor with her feet tucked under her and this measurement is taken to be the diameter of her body, found to be 17 cm. For tail measurements, I held her tail out and measured the underside of her tail up to where her tail attaches to her body and found her tail to be 27 cm long. For tail weight, I found a dead cat (see endnote) with a similar weight and same tail length and I cut the tail off and it weighed 70 grams. The head was measured from the bottom of the chin to the top of the head by holding a ruler along the side of the head and the diameter was taken to be 6 cm. The weight of the head was estimated to be 250 grams (see endnote). The weight of the upper body was estimated to be 2.74 kg and the weight of the lower body was estimated to be 1.7 kg. The upper body was estimated to weigh more because the head, ribs and most of the organs are in the upper half of the body and so there is more weight in the upper half of the body.

For the calculations to work, there must be an assumption that there is no change in inertia throughout the rotations for either of the rotating bodies. So all the models presented in this study are rigid cylinders or cones and the weight of the cat is never redistributed throughout the course of motion.

Figure 1



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first model proposed, the blue conical block represents the tail segment and the maroon cylinder represents the body and the head as one rotating segment. The tail is assumed to rotate around the axis at 90° to the body so that the length l is the length used in calculating inertia on the tail and length r is used to calculate the inertia of the body and head. The calculations are in Appendix 1.

Figure 2

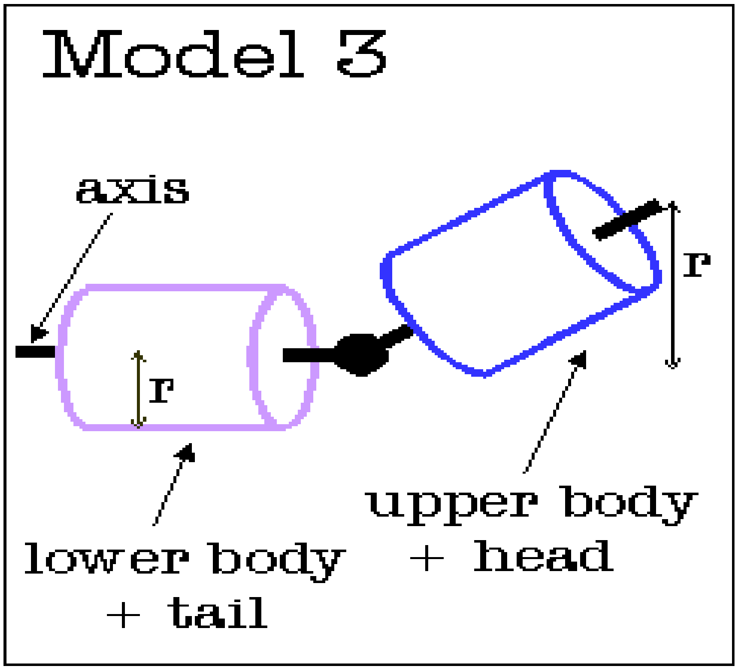

Figure 2 shows the second model proposed. The smaller green cylinder represents the head as the first segment and the large orange cylinder represents the body and the tail as one segment. The head and the body rotates around the axis in the same manner and the lengths used in calculating inertia is indicated by r in both segments. Biologically, the head can only rotate 180°, however this is ignored in the calculations and will be discussed further in the discussion. The calculations for this model are in Appendix 2.

Figure 3

Figure 3 shows the third model proposed. The purple cylinder represents the first segment consisting of the lower body and the tail and the second segment comprising the upper body and the head is represented by the dark blue cylinder. The upper body and the head is seen to rotate at an angle away from the body in a circular pattern. This is similar to the rotation of the tail in model 1 but the tail is rotating at 90° to the body and in this model that angle is larger. The lengths used to calculate inertia is shown on the figure is shown to be r. The constant in calculating inertia for this model must be a number between 1/2½to 1/10, this constant is dependant on the path of rotation and I estimate it to be 1/6. The r is estimated to be about 12 cm observed from the video recording. The calculations for this model is in Appendix 3.

A digital camera recorded my cat falling and righting herself and these videos were saved onto a disk. My cat was held in my hands with her feet pointing toward the ceiling and released by sliding my hand out sideways. Wong Wong righted herself with different levels of success. Overall, 4 videos were shot.

The calculations give us a number for the angular velocity at which the rotating body part would have to move at to rotate the cat’s body 180° and it also gives us an estimate as to how many rotations of the moving body part are needed. This combined with a video I took of my cat, should give an idea of whether or not a cat uses any of the models presented.

Results

Model 1 calculations show that the angular velocity would have to be too high (331 rad/s) and you have to rotate the tail too many times. In Model 2, the angular velocity is ridiculously high (1141 rad/s) and it would be impossible to move that fast. Model 3 gives reasonable and technically executable numbers for angular velocity (5 rad/s) and number of turns (about 288 degrees). Results are summed up in table 1.

The video tape shows that my cat is able to right herself in a very short time and also she does so in one rotation but it also shows that she does not land on her feet all of the time. She fell once even though she apparently had plenty of time to turn over during her descent. In several of the shots, she wriggles in my hands before I release her and that motion resembles the motion she makes when she rotates in the air.

The calculations and the motions from the tape together seem to suggest that model three could be used to model how a cat moves while it is falling to right itself.

Table 1

	Model 1	Model 2	Model 3
Number turns required	**16**	**55**	**.8**
Angular velocity	**331 rad/s**	**1141 rad/s**	**5 rad/s**

Table 1 shows the number of rotations that the moving body part of the cat would have to make in order for the cat's body to rotate 180° and land feet down if it falls in an upside down position. It also shows the speed at which that body part must rotate in order to rotate its body in 0.3 seconds. Model 2 is has the fastest and the most number of rotations needed, and model 3 has the slowest and the lowest number of rotations needed.

Discussion

Model 1 does not seem to be the mechanism used because the cat would have to move its tail really fast and even if it could do that, it would have to rotate its tail more then once. The video of the cat falling does not seem to concur with this model. The cat's tail in the video seems to rotate rather slowly and does not make more then two rotations.

Model 2 cannot be the mechanism used because it is impossible for the cat's head to move that fast. Even if speed does is not a factor, the cat would have to rotate it head completely around 55 times in order to right itself. Biologically, the head is restricted in movement to about 180° and would twist off should it rotate much more then this. Thus the second model is insufficient in modeling how a cat moves in the air. The videos seem to show also that the head is not much involved in the turning of the cat’s body as it dose not seem to move very much during falling.

The two segment, one jointed system in model 3 seems to suggest that this model is a possible explanation as to how a cat rights itself in the air. The video seems to confirm the mechanism used to in model 3 where the cat rotates its body about 3/4 of a circle at about 5 radians per second.

Model 3 worked because it had a large inertia. The conservation of angular momentum really only has two variables. One is inertia and the other is angular velocity or speed. If the inertia is large on one side of the equation and small on the other, the speed for one must then make up the difference in order to match the other. In model 1 and 2 the inertia was simply too small for the head or tail to move the large mass of the body. Because of the restriction of two variables, the speed had to be large where the mass and inertia was small. So impossibly large angular velocities resulted from calculations in models 1 and 2. In model 3, the angular momentum was smaller because the two body parts of the cat were more similar and the inertia of both were similar so that the smaller differences in inertia resulted in a smaller calculated speed.

The assumption that was held throughout the project was that a cat rights itself in 0.3 seconds. This only an estimate, of course the longer the cat actually takes while righting itself, the smaller the angular velocity will have to be. However because the speeds for models 1 and 2 are both so high, the time would not make a difference in overall results unless the time was extended to an improbably long length of time.

Another assumption made was that the legs of the cat is pulled in during the entire rotation of the cat. The video does not support this. In fact the cat clearly extends and pulls in its legs during the rotation. When the legs are extended it will make r bigger and will create an even bigger disparity in inertia for models 1 and 2 and thus make the calculated results even more improbable. However, for model 3 the extension of the cat's legs could increase r for the rotating half of its body and thus create a larger inertia for the moving segment therefore lowering angular velocity and rotation distance. Further study could be conducted here to see how much leg extension affects a cat's movements in the air.

The pivotal assumption made in this project is the one that segments a cat into two rigid, unchanging bodies that falls through the air. A cat is nothing like this. It has muscles and tendons that can possibly collectively place an equivalent of a small external torque on the body because a body can stretch and therefore one segment may be able to move after the other one has stopped moving and through use of muscles a cat can force the body to stay where it is to change positions. Also the joints of a cat is not rigid, it can move in many directions thus changing the inertia and so this assumption could cause great differences in the results obtained by the calculations. Since this assumption makes the results ambiguous, the results cannot be justified and so the real mechanism used by cats remains inconclusive.

Several question arose while doing this project. The first pertains to the head. It is very probable from these calculations that head movement does not at all contribute to the righting mechanism of a cat in the air. Perhaps further study could be done to explore what the role of the head is. The cat could be turning to spot the ground and does this have any affect as to the time in which it is able to turn itself? Or perhaps does knowing that the ground is closer help motivate the cat to turn faster?

Another question that could be interesting to explore is whether or not the cat is using more then a two segment system. The simplest model is a two segment system but in all likelihood, many other parts of a cat contributes to its movement and so perhaps study could be done to determine if a multijointed and segmented cat would be better then the model proposed here. For instance the cat could use some combination of model 1 and model 3. Perhaps dropping tail-less cats and timing them to see if there is a time difference between them and cats with tails.

One question raised solely by the video was why my cat was not able to land on her feet during the first fall. She seemed to have lots of time and space but she landed on her side. Could the reason for her failure be that she was not prepared to fall? This would suggest that righting is not a instinctive motion. Perhaps a study could be done there. Could the reason be that she was stretched out and so in order for her to increase her angle of rotation and thus increase r she would have to move up against gravity? An experiment would be to release cats over a soft area but with their bodies extended to see if they are able to land on their feet.

Acknowledgements

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Endnote

No unnecessary or intentional harm was inflicted upon my cat at any time during the experiments done for this project. The dead cat was one used for dissection by the UBC vertebrate biology 204 class and the cat was selected to match the size and length of Wong Wong. The dead cat was weighed to be about 5 kg with a plastic bag wrapped around it and the tail was measured to be 27 cm and 70 grams. Both the cat and its tail held some excess water due to preservatives but the cats were cut open and so lost a lot of fluid as well.

The heads of the cats are used by the class for dissection and so I could not cut off the head for measurements.

Appendix 1

$$I_{(body)}\acute{\omega}_{(body)} = I_{(tail)}\acute{\omega}_{(tail)}$$

$$I_{(tail)}=0.1*mass*length^2$$
$$=0.1*0.07kg*0.27m^2$$
$$= 0.00051kgm^2$$

$$I_{(body)}=0.5*m*r^2$$
$$=0.5*4.5kg*0.085m^2$$
$$=0.016kgm^2$$

$$\acute{\omega}_{(body)}=\phi/time$$
$$=\pi/0.3s$$
$$=10.47\ rad/s$$

$$\acute{\omega}_{(tail)}=I_{(body)}\ \acute{\omega}_{(body)}/I_{(tail)}$$
$$=10.47rad/s*0.016kgm^2/0.00051kgm^2$$
$$=331\ rad/s$$

$$\phi = \acute{\omega}_{(tail)}*time$$
$$=331rad/s * 0.3\ s$$
$$=99\ rad$$

$$N\ turns = \phi/2\pi$$
$$=99\ rad/2\pi$$
$$=16$$

Appendix 2

$$I_{(body)}\acute{\omega}_{(body)} = I_{(head)}\acute{\omega}_{(head)}$$

$$I_{(head)}=0.5*mass*length^2$$
$$=0.5*0.25kg*0.03m^2$$
$$= 0.00011kgm^2$$

$$I_{(body)}=0.5*m*r^2$$
$$=0.5*4.3kg*0.085m^2$$
$$=0.015kgm^2$$

$$\acute{\omega}_{(body)}=\phi/time$$
$$=\pi/0.3s$$
$$=10.47rad/s$$

$$w_{(tail)}=Iw_{(body)}/I_{(tail)}$$
$$=10.47\ rad/s*0.015kgm^2/0.00011kgm^2$$
$$=1141\ rad/s$$

$$\phi = \acute{\omega}_{(head)}*time$$
$$=1141rad/s * 0.3\ s$$
$$=342\ rad$$

$$N\ turns = \phi/2\pi$$
$$=342\ rad/2\pi$$
$$=55$$

Appendix 3

$$I_{(upper\ body)}\acute{\omega}_{(upper\ body)} = I_{(lower\ body)}\acute{\omega}_{(lower\ body)}$$

$$I_{(lower\ body)} = 0.5*mass*length^2$$
$$= 0.5*1.7kg*0.085m^2$$
$$= 0.0098kgm^2$$

$$I_{(upper\ body)} = 1/6*m*r^2$$
$$= 1/6*2.7kg*0.12m^2$$
$$= 0.0065kgm^2$$

$$\acute{\omega}_{(body)} = \phi/time$$
$$= \pi/0.3s$$
$$= 10.47rad/s$$

$$\acute{\omega}_{(lower\ body)} = I_{(upper\ body)}\acute{\omega}_{(upper\ body)}/I_{(lower\ body)}$$
$$= 10.47\ rad/s*0.0065kgm^2/0.0098kgm^2$$
$$= 16\ rad/s$$

$$\phi = \acute{\omega}_{(head)}*time$$
$$= 16rad/s * 0.3\ s$$
$$= 5\ rad$$

$$N\ turns = \phi/2\pi$$
$$= 5\ rad/2\pi$$
$$= 0.8$$