Linear Superposition of Waves

The overall amplitude $A(x,t)$ at a given time and place is just the **sum** of the amplitudes $A_i(x,t)$ of independently propagating waves.

For **two** waves,

$$A_1 \exp[\, i\,(k_1 x - \omega_1 t + \varphi_1)\,] \quad \text{and} \quad A_2 \exp[\, i\,(k_2 x - \omega_2 t + \varphi_2)\,]\,,$$

$$A(x,t) = A_1\, e^{\,i\theta_1} + A_2\, e^{\,i\theta_2}$$

$$\text{where} \quad \theta_1 = k_1 x - \omega_1 t + \varphi_1 \quad \textit{and} \quad \theta_2 = k_2 x - \omega_2 t + \varphi_2\,.$$

This is boring unless θ_1 differs from θ_2. There are 2 ways this happens:

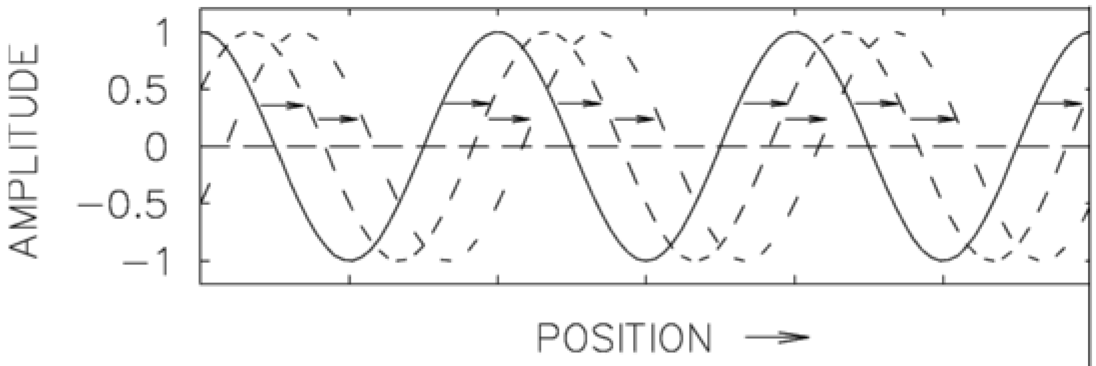
- **Frequency Differences**: **beats** $(\omega_1 \approx \omega_2)$ or **standing waves** $(\omega_1 = -\omega_2)$
- **Phase Differences**: $(\varphi_1 \neq \varphi_2)$ which may have various causes.

$\Delta\theta \equiv \theta_1 - \theta_2 = 2\pi n$ gives **constructive** interference.

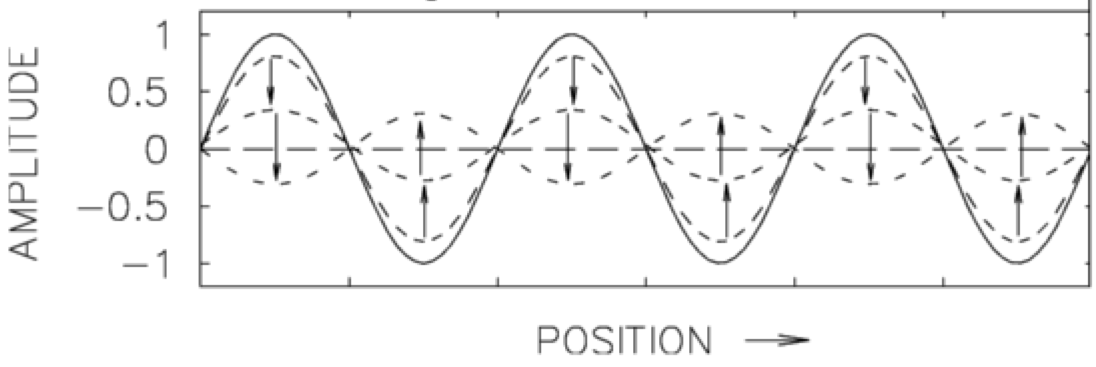
$\Delta\theta = \pi(2n+1)$ gives **destructive** interference.

Standing Waves

Traveling Wave at successive times:

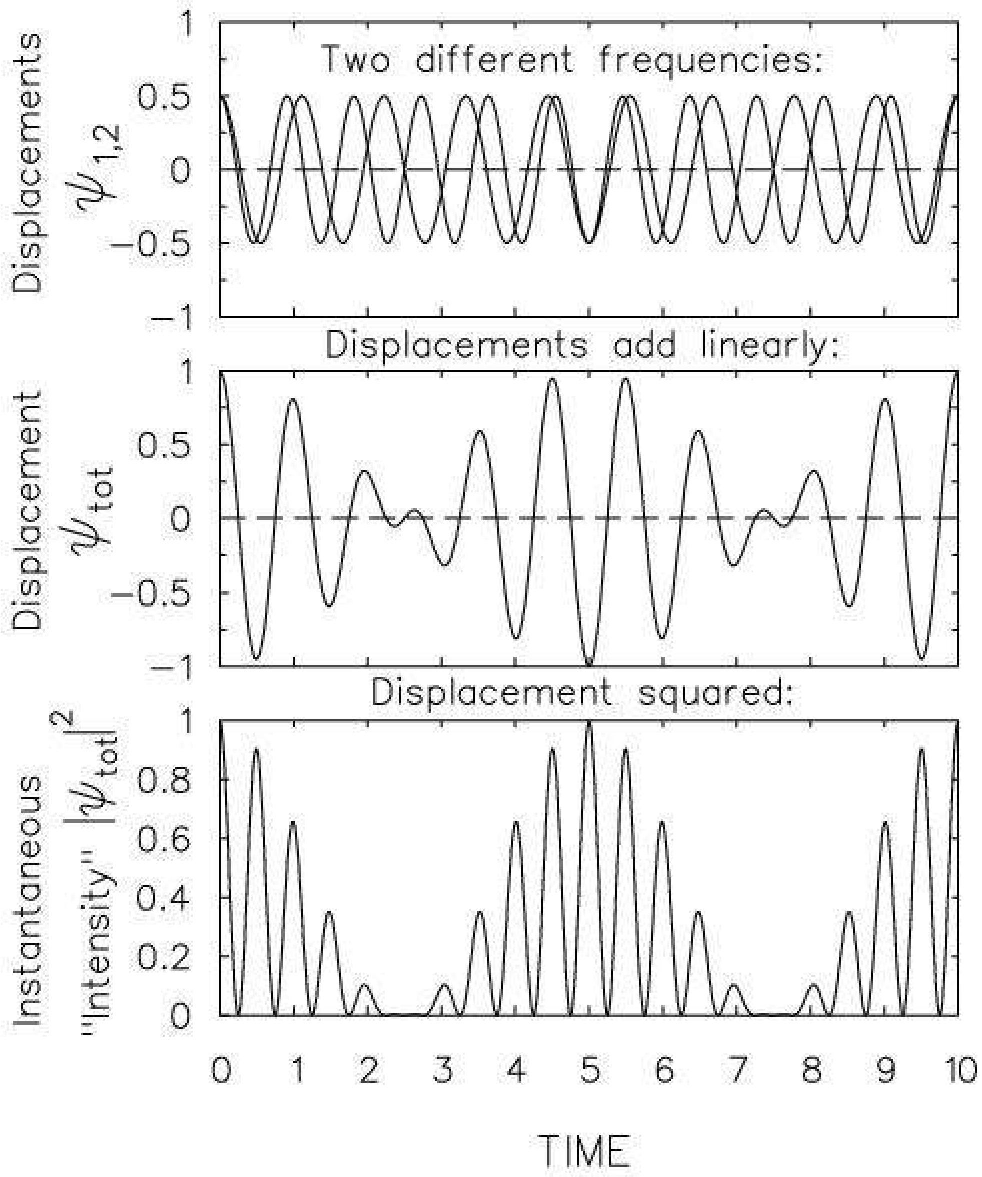


Standing Wave at successive times:



Sum of two equal-amplitude waves of the same frequency and wavelength **traveling in opposite directions** ($\omega_1 = -\omega_2$).

Beats



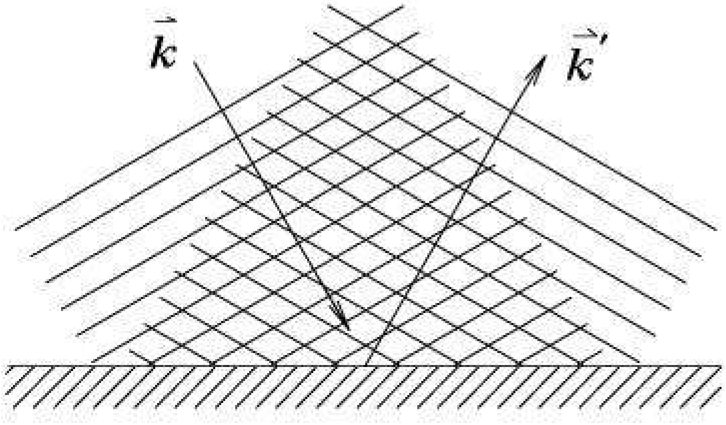
$$\psi(t) = A\,[e^{i\omega_1 t} + e^{i\omega_2 t}]$$

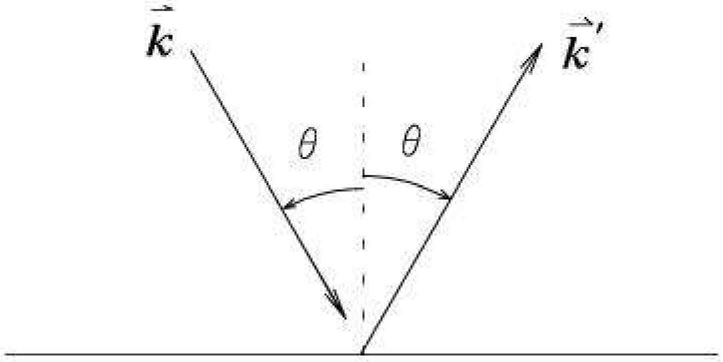
$$\omega_1 = \omega + \Omega, \quad \omega_2 = \omega - \Omega$$

$$\psi(t) = [2A\cos\Omega t]\,e^{i\omega t}$$

Normally we perceive "the **intensity**" as the **time average** of the square of the instantaneous amplitude.

Reflection



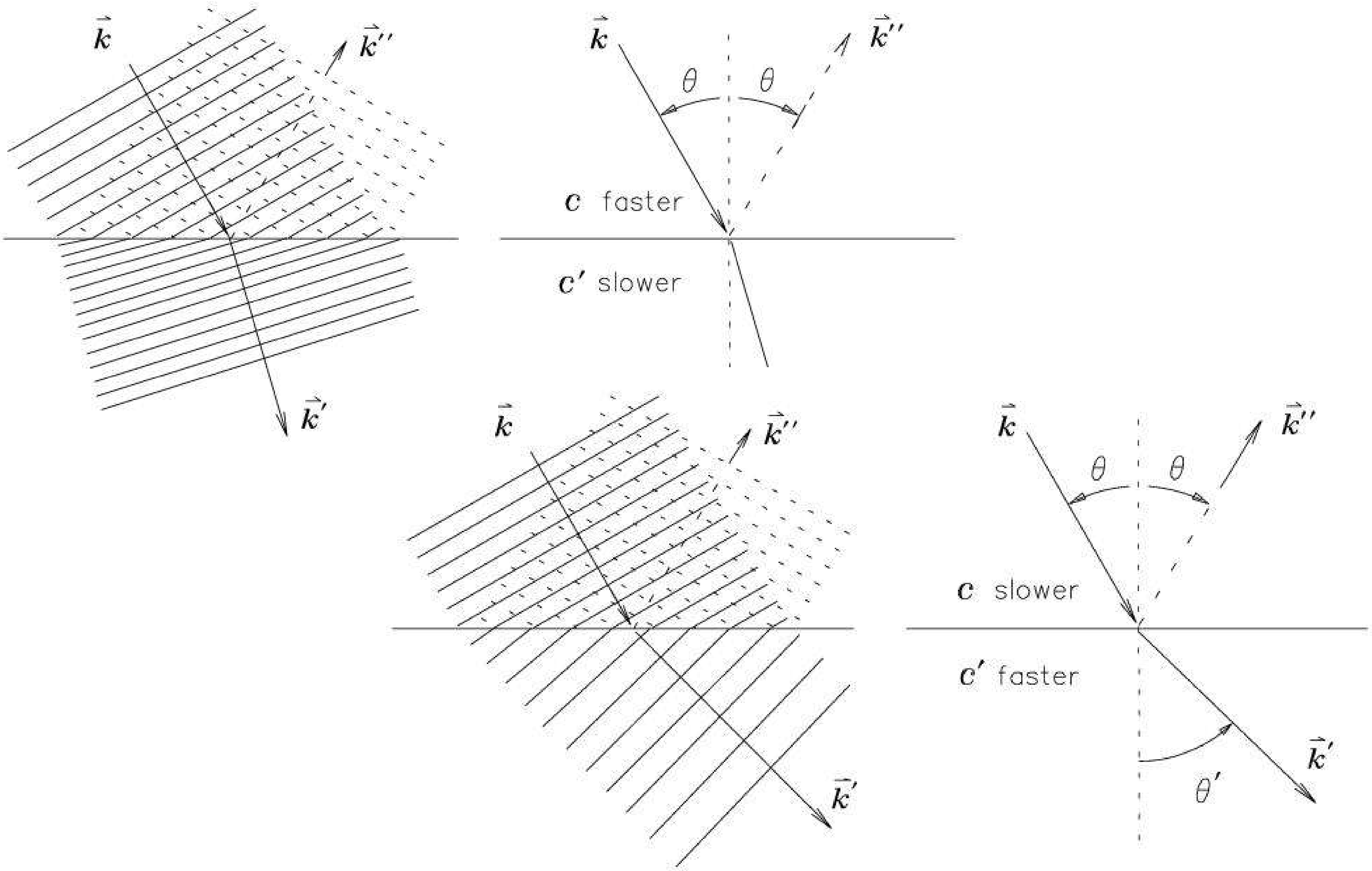


Note: reflection **always** occurs at **any** interface between two media.

Reflection off a **denser** medium causes a **phase reversal**: $\Delta\varphi = \pi$

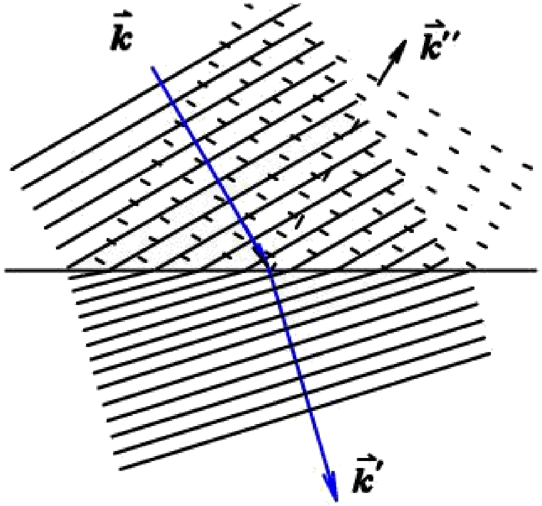
Reflection off a **less dense** medium causes **none**: $\Delta\varphi = 0$

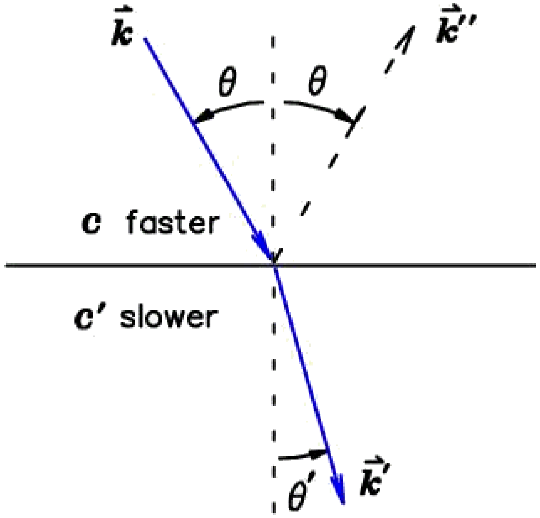
Refraction



$\vec{k}$
$\vec{k}''$
$\vec{k}'$
$\vec{k}$
$\vec{k}''$
θ
θ
c faster
c' slower
$\vec{k}$
$\vec{k}''$
$\vec{k}'$
$\vec{k}$
$\vec{k}''$
θ
θ
c slower
c' faster
$\vec{k}'$
θ'

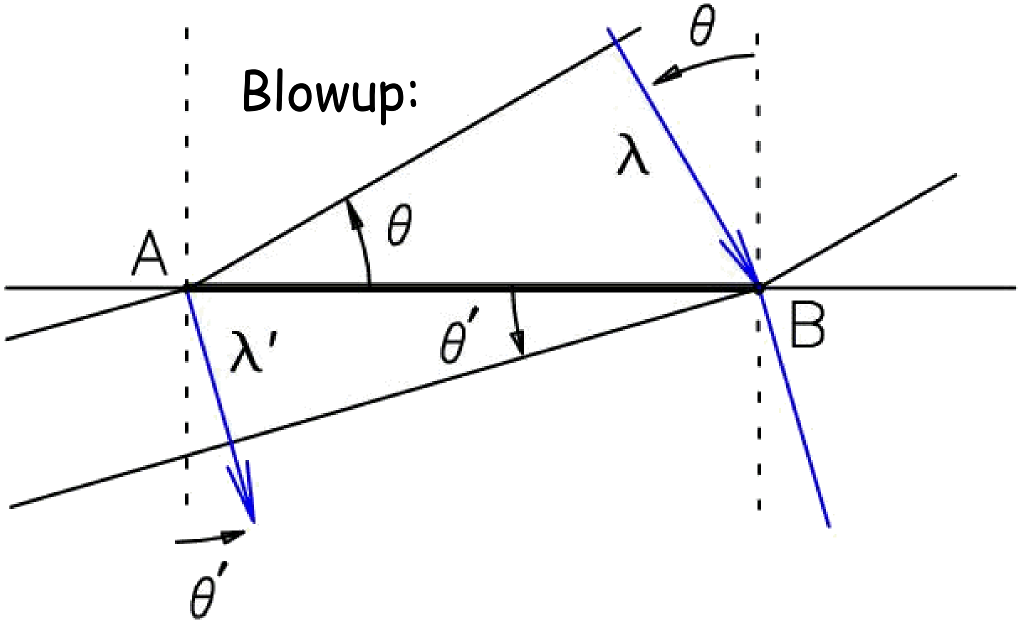
Snell's Law





Consider the case where the wave enters a "denser" medium (one where it propagates slower): define the index of refraction

$$n \equiv c/c' \geq 1.$$



The line AB is the hypoteneuse of both right triangles:

$\lambda = \text{AB} \sin\theta$ and $\lambda' = \text{AB} \sin\theta'$

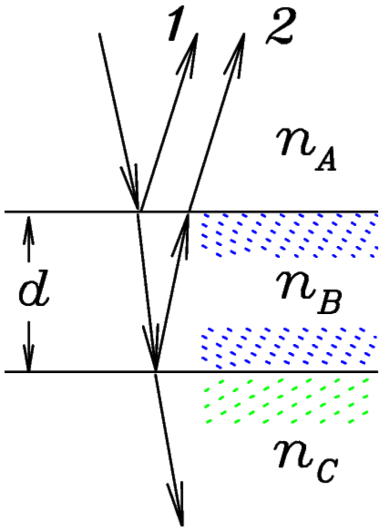
so $\lambda/\lambda' = \sin\theta/\sin\theta'$ or,

since $\lambda/\lambda' = c/c' \equiv n$,

$$\boxed{\sin\theta/\sin\theta' = c/c' \equiv n}\,.$$

Thin Film Interference

We always draw the reflected and refracted rays at a small angle to the normal so that the two reflected rays (*1* & *2*) can be shown separately; but in reality we are always talking about **normal incidence**.



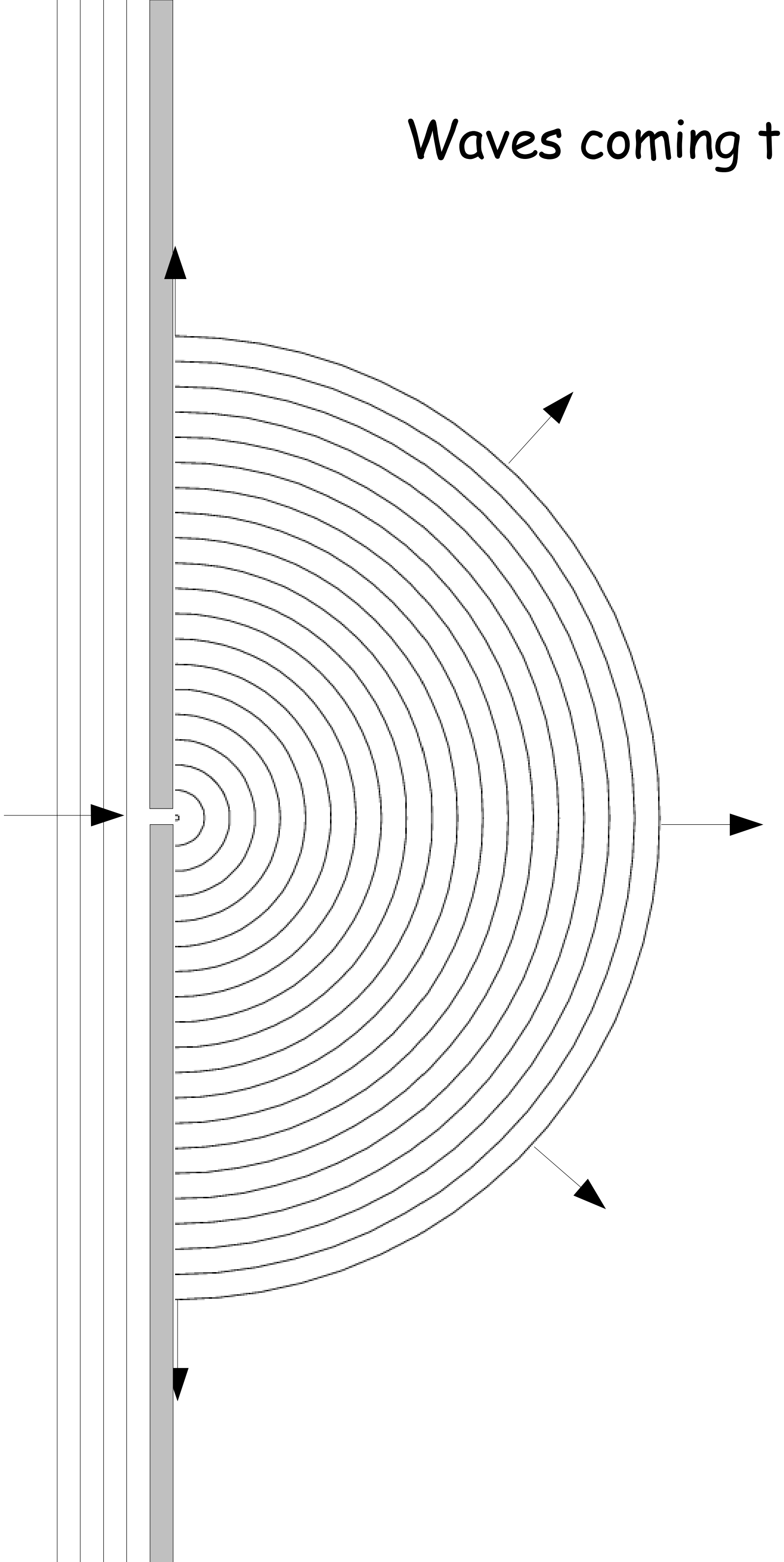
To decide if rays *1* & *2* are **in phase** or **out of phase**, we **add up their phase differences**. Upon **reflection**, if $n_B > n_A$, ray *1* experiences a phase shift of π; ray *2* has a similar phase shift if $n_C > n_B$. Then the path length difference (2d) gives a phase difference of $\Delta\theta_{\text{path}} = 2\pi(\Delta\ell/\lambda_B)$ where λ_B is the wavelength in medium B. Let's suppose $n_C > n_B > n_A$ so that both reflected rays get the same "phase flip". Then the path length difference of $2d$ is the only source of $\Delta\theta = 2\pi(2d/\lambda_B)$.

If $d = \lambda_B/4$ (what we call a "**quarter-wave plate**") then rays *1* & *2* will interfere **destructively**, giving **minimum reflection** & **maximum transmission**. This is used in "anti-glare" coatings on windows, glasses and camera lenses.

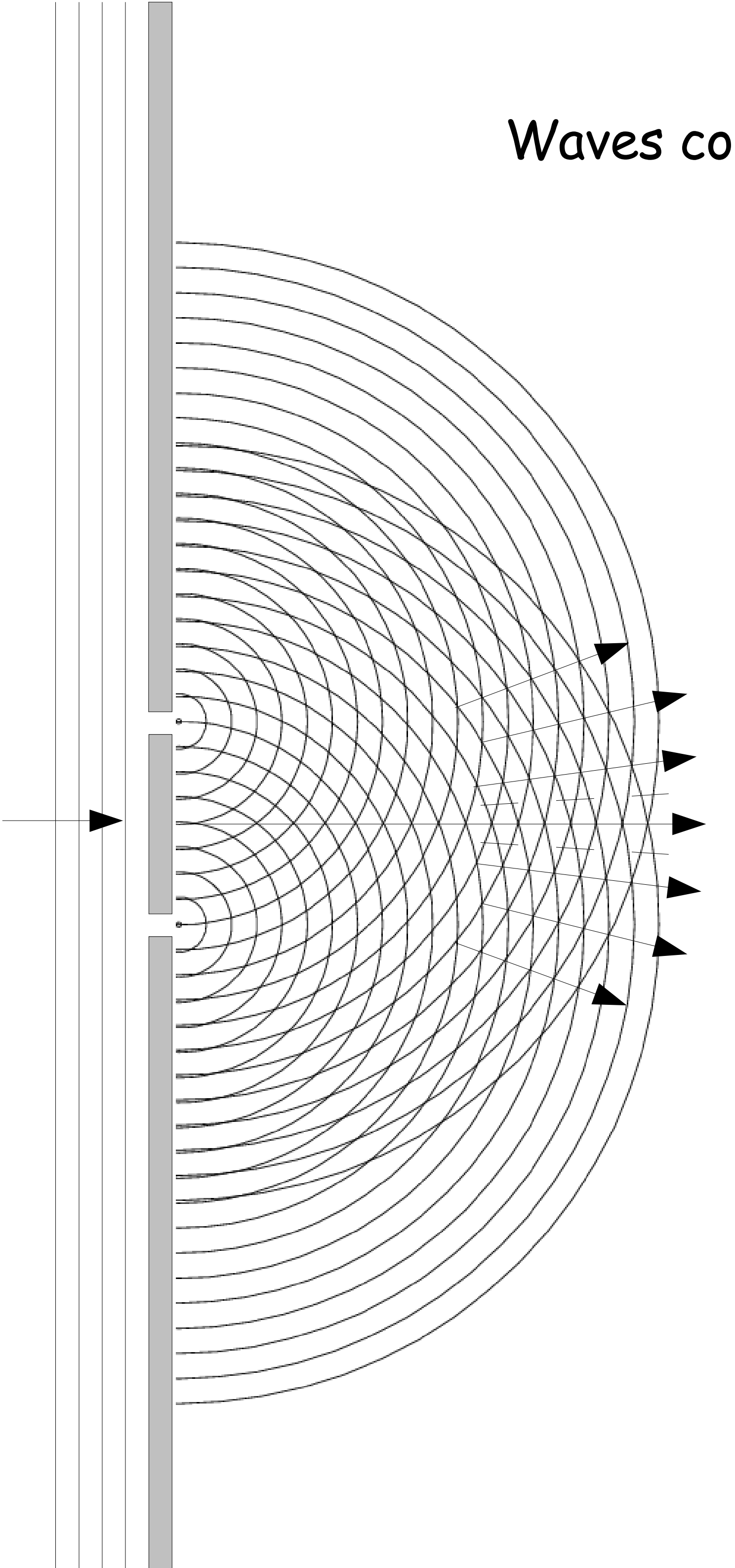
HUYGENS' PRINCIPLE:

"All points on a wavefront can be considered as *point sources* for the production of *spherical secondary wavelets*. At a later time, the new position of the wavefront will be the *surface of tangency* to these secondary wavelets."

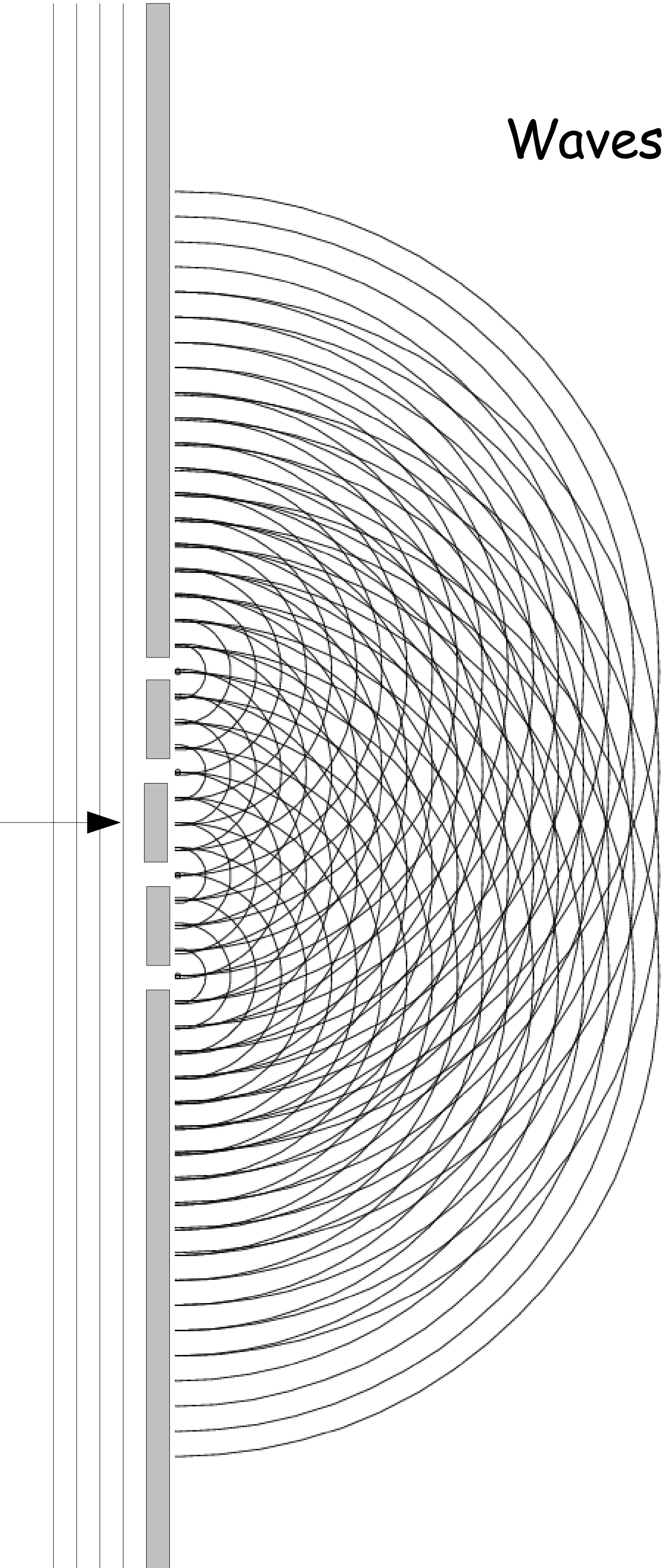
Waves coming through a small gap in a seawall:



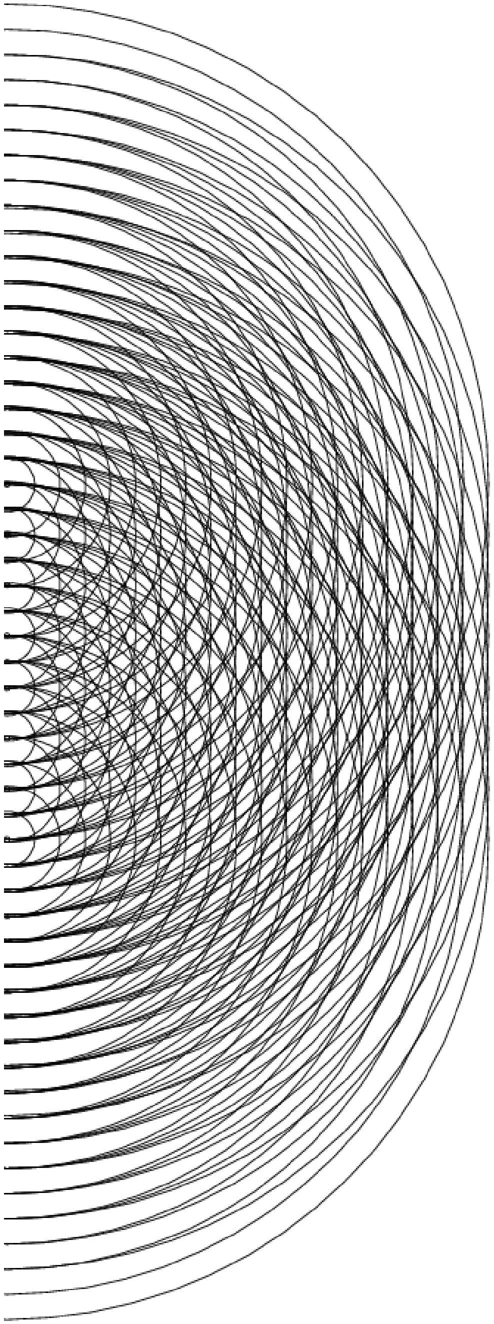
Waves coming through 2 small gaps in a seawall:



Waves coming through 4 small gaps in a seawall:



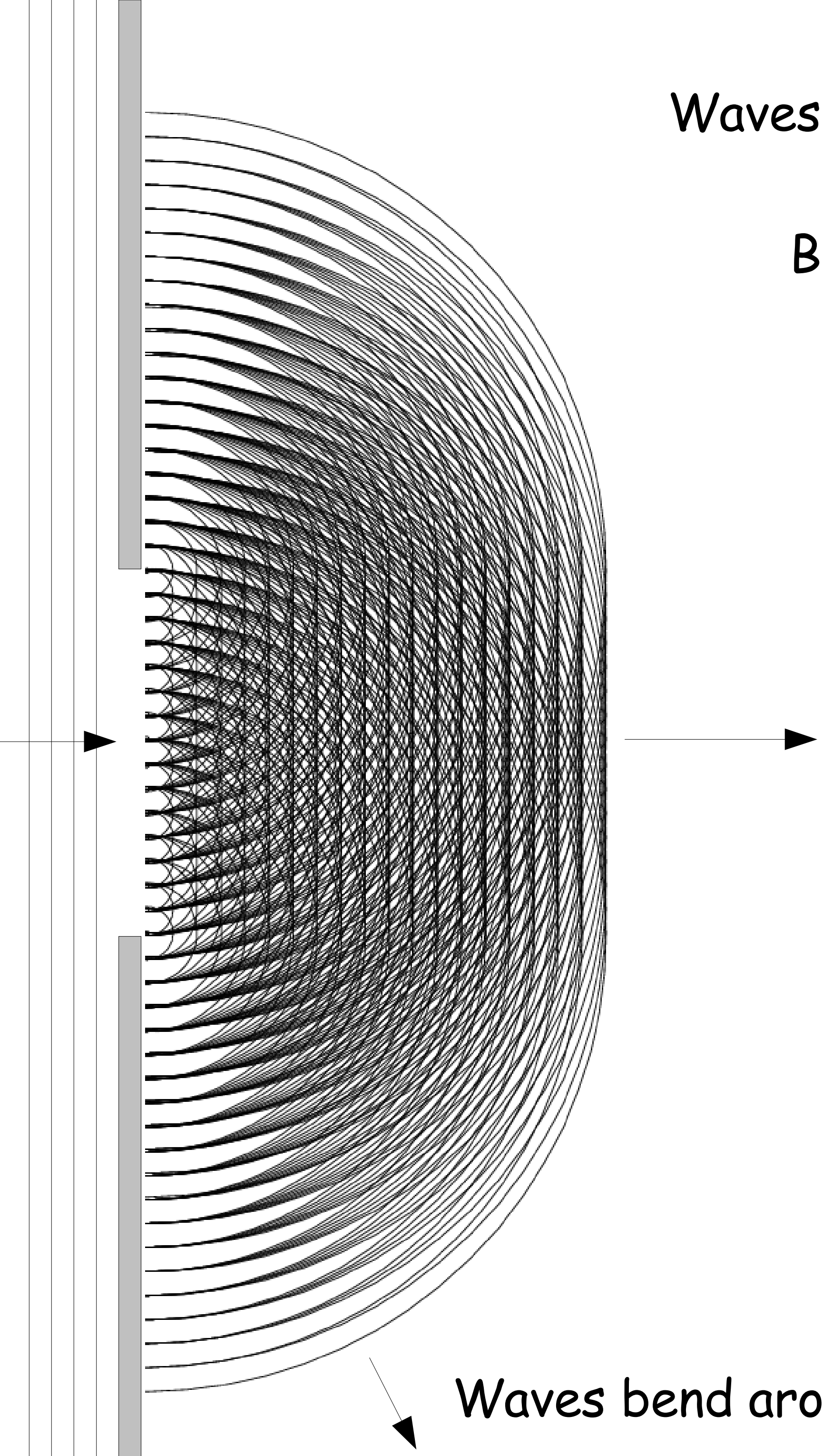
Waves coming through 8 small gaps in a seawall (not shown):



Waves coming through 16 small gaps in a seawall:

Begins to look more like **one big gap**....

(This is what we call "**diffraction**".)

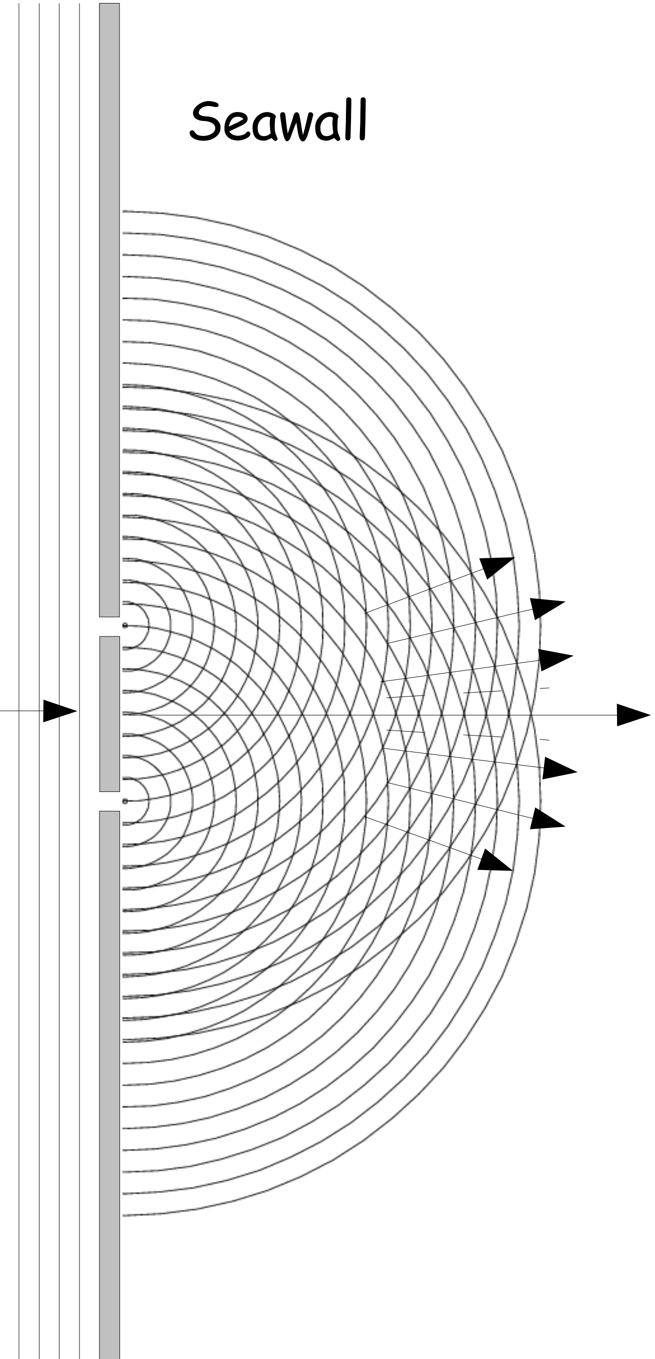


"Central Maximum"

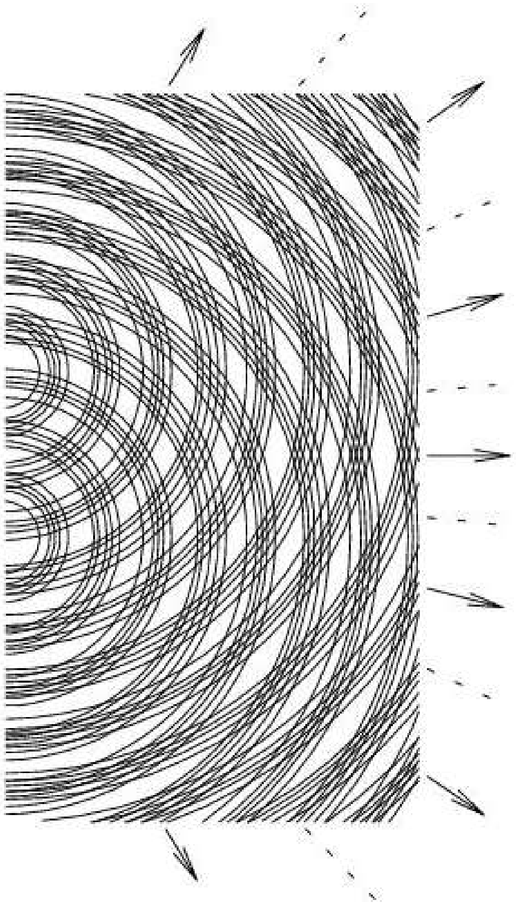
Waves bend around corners !

Two-Slit Interference

Seawall



Young's diagram



"Ray" formulation