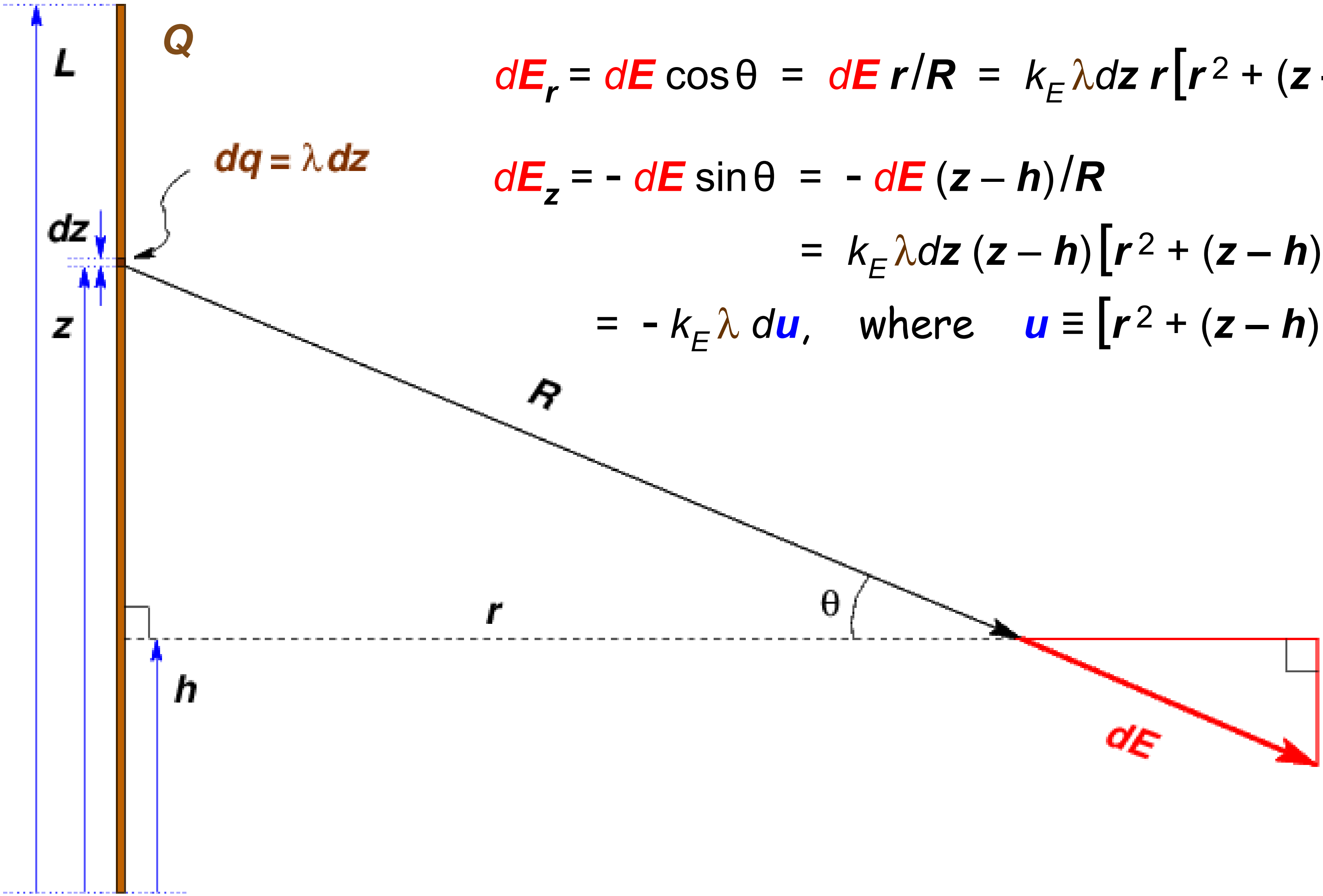
Finite Rod of Charge – general case

$\lambda = Q/L$



$$dE = k_E\, dq/R^2 = k_E \lambda dz \left[r^2 + (z - h)^2\right]^{-1}$$

$$dE_r = dE \cos\theta \;=\; dE\, r/R \;=\; k_E \lambda dz\, r\left[r^2 + (z-h)^2\right]^{-3/2}$$

$$dE_z = -\,dE \sin\theta \;=\; -\,dE\,(z-h)/R$$

$$= \; k_E \lambda dz\,(z-h)\left[r^2 + (z-h)^2\right]^{-3/2}$$

$$= \; -\,k_E \lambda\, du, \quad \text{where} \quad u \equiv \left[r^2 + (z-h)^2\right]^{-1/2}$$

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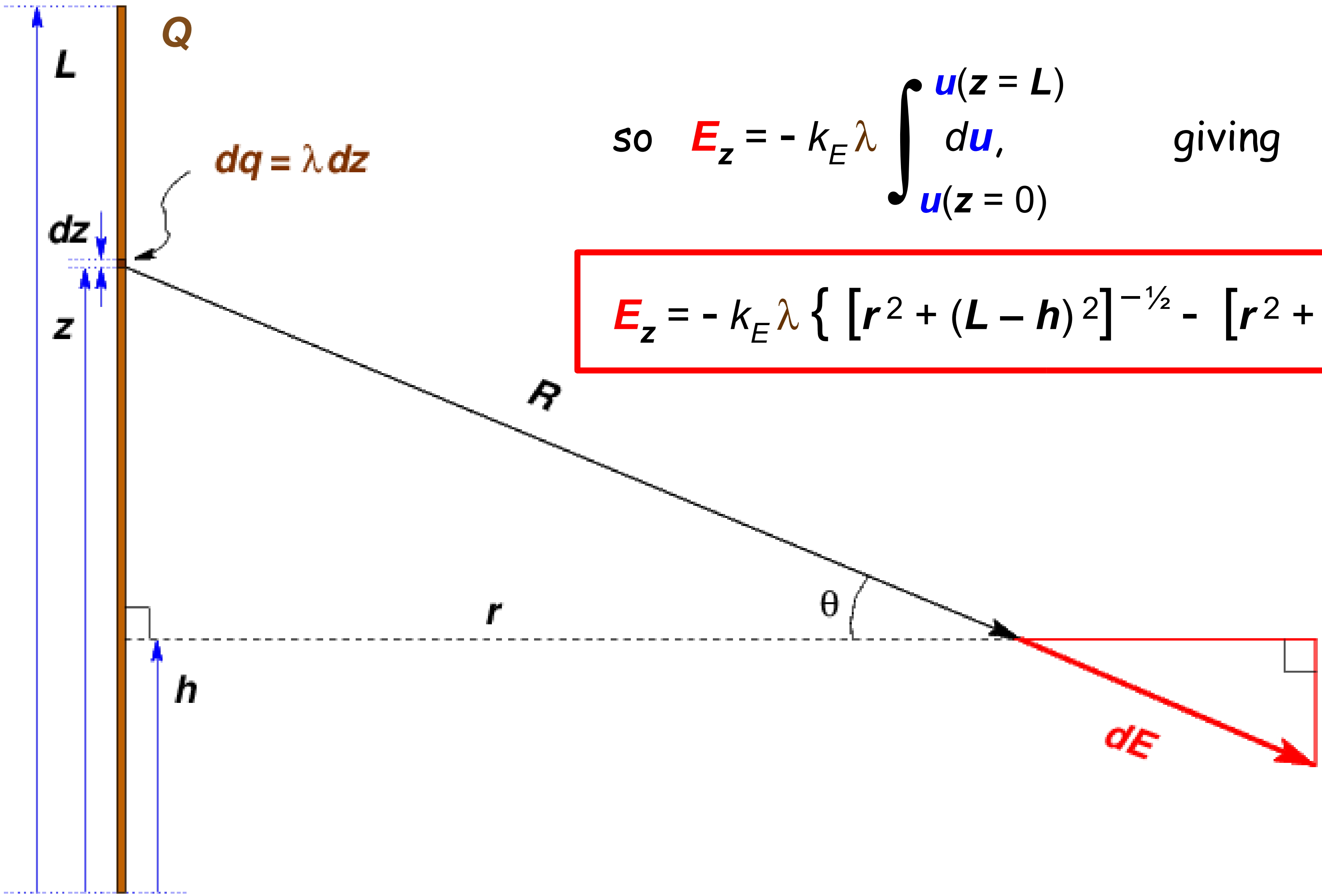
$dE_z = -\, k_E \lambda \, du$, where $u \equiv \left[r^2 + (z - h)^2\right]^{-1/2}$

so $E_z = -\, k_E \lambda \displaystyle\int_{u(z=0)}^{u(z=L)} du$, giving

$$E_z = -\, k_E \lambda \left\{ \left[r^2 + (L - h)^2\right]^{-1/2} - \left[r^2 + h^2\right]^{-1/2} \right\}$$



Finite Rod of Charge – general case

$\lambda = Q/L$

$$dE_r = dE\cos\theta = (k_E \lambda dz/R^2)(r/R) = k_E \lambda dz\, r\left[r^2 + (z-h)^2\right]^{-3/2}$$

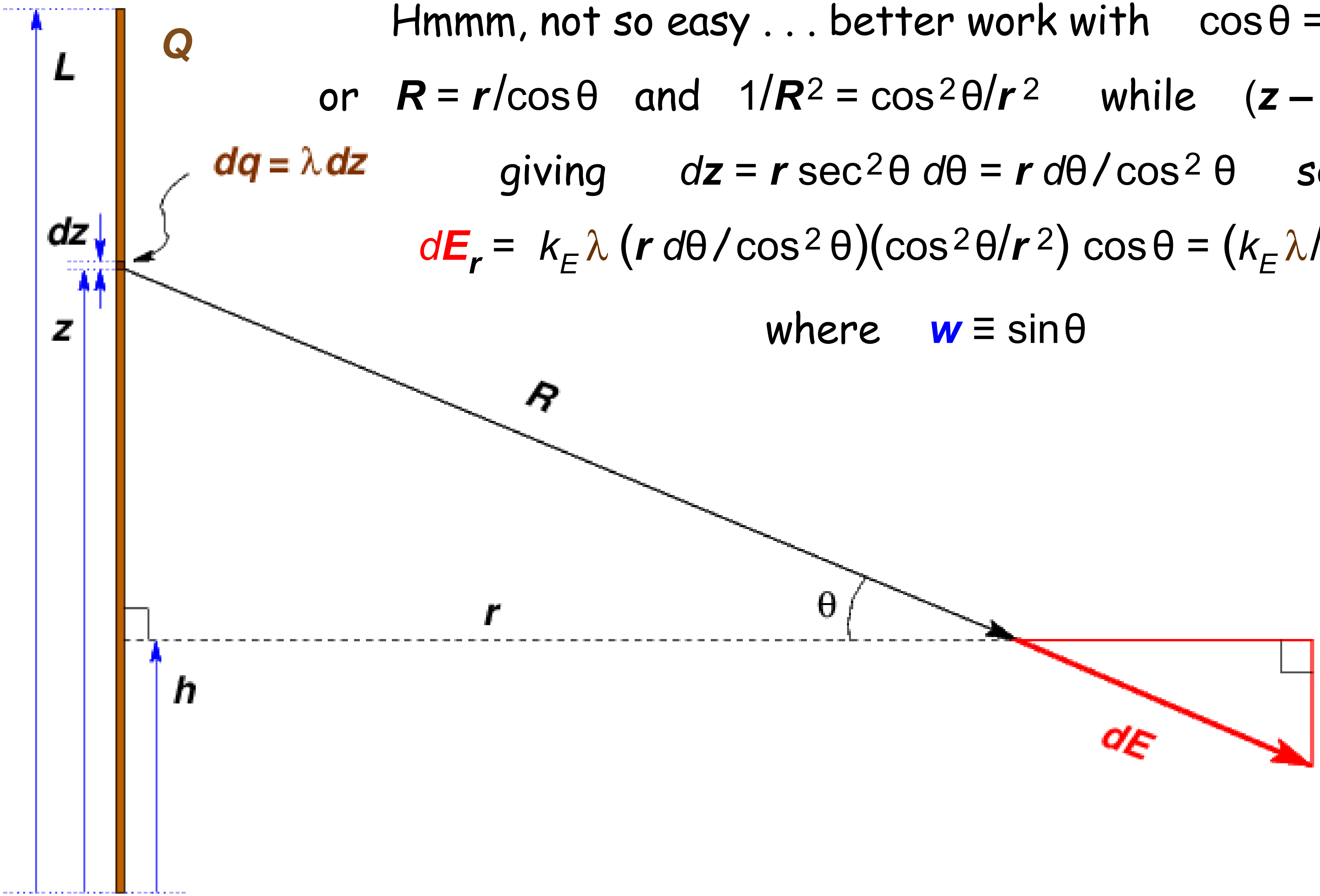
Hmmm, not so easy . . . better work with $\cos\theta = r/R$

or $R = r/\cos\theta$ and $1/R^2 = \cos^2\theta/r^2$ while $(z-h) = r\tan\theta$,

giving $dz = r\sec^2\theta\, d\theta = r\, d\theta/\cos^2\theta$ so that

$$dE_r = k_E \lambda\,(r\, d\theta/\cos^2\theta)(\cos^2\theta/r^2)\cos\theta = (k_E \lambda/r)\, dw,$$

where $w \equiv \sin\theta$



Finite Rod of Charge – general case

$\lambda = Q/L$

$$dE_r = k_E \lambda \, (r \, d\theta / \cos^2\theta)(\cos^2\theta / r^2) \cos\theta = (k_E \lambda / r) \, dw,$$

where $w \equiv \sin\theta$, giving

$$E_r = (k_E \lambda / r) \int_{w(z=0)}^{w(z=L)} dw, \qquad \text{or}$$

$$\boxed{E_r = (k_E \lambda / r) \left\{ (L-h)\left[r^2 + (L-h)^2\right]^{-1/2} + h\left[r^2 + h^2\right]^{-1/2} \right\}}$$