

Finite Rod of Charge - general case

$\lambda = Q/L$

$d\mathbf{E} = k_E dq/R^2 = k_E \lambda dz [r^2 + (z-h)^2]^{-1}$

$dE_r = dE \cos \theta = dE r/R = k_E \lambda dz r [r^2 + (z-h)^2]^{-3/2}$

$dE_z = -dE \sin \theta = -dE (z-h)/R$
 $= k_E \lambda dz (z-h) [r^2 + (z-h)^2]^{-3/2}$
 $= -k_E \lambda du, \text{ where } u \equiv [r^2 + (z-h)^2]^{-1/2}$

The diagram illustrates the geometry for calculating the electric field from a finite rod of length L and total charge Q . A small charge element $dq = \lambda dz$ is located at a distance z from the top of the rod. A point is located at a horizontal distance r from the rod and a vertical distance h from the bottom of the rod. The distance from the charge element to the point is R , and the angle between the line R and the horizontal dashed line is θ . A red vector $d\mathbf{E}$ is shown pointing from the charge element to the point, with a right-angle symbol at the point.

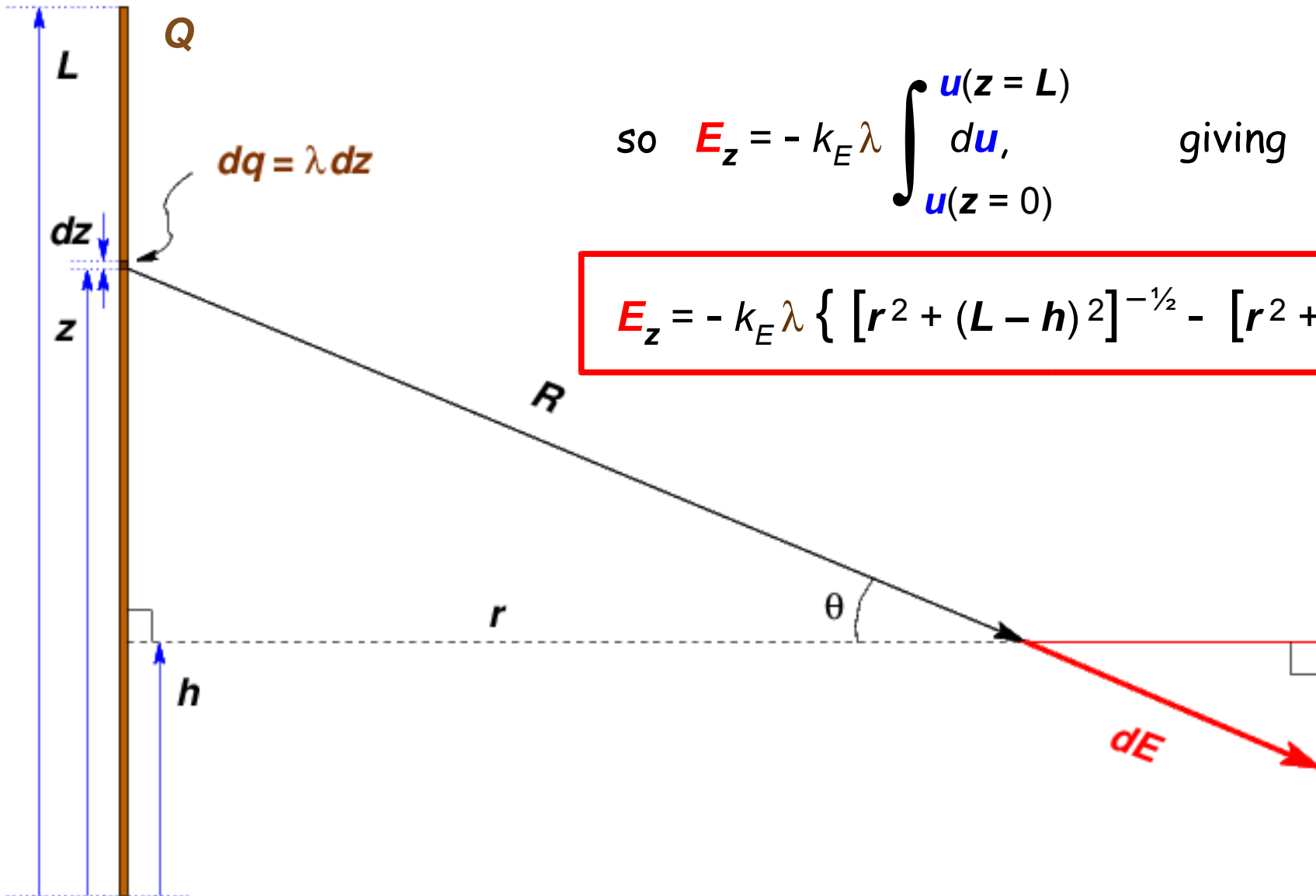
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$$d\mathbf{E}_z = -k_E \lambda du, \quad \text{where} \quad u \equiv [r^2 + (z-h)^2]^{-1/2}$$

$$\text{so } \mathbf{E}_z = -k_E \lambda \int_{u(z=0)}^{u(z=L)} du, \quad \text{giving}$$

$$\mathbf{E}_z = -k_E \lambda \left\{ [r^2 + (L-h)^2]^{-1/2} - [r^2 + h^2]^{-1/2} \right\}$$



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$$dE_r = dE \cos \theta = (k_E \lambda dz / R^2)(r/R) = k_E \lambda dz r [r^2 + (z-h)^2]^{-3/2}$$

Hmmm, not so easy ... better work with $\cos \theta = r/R$

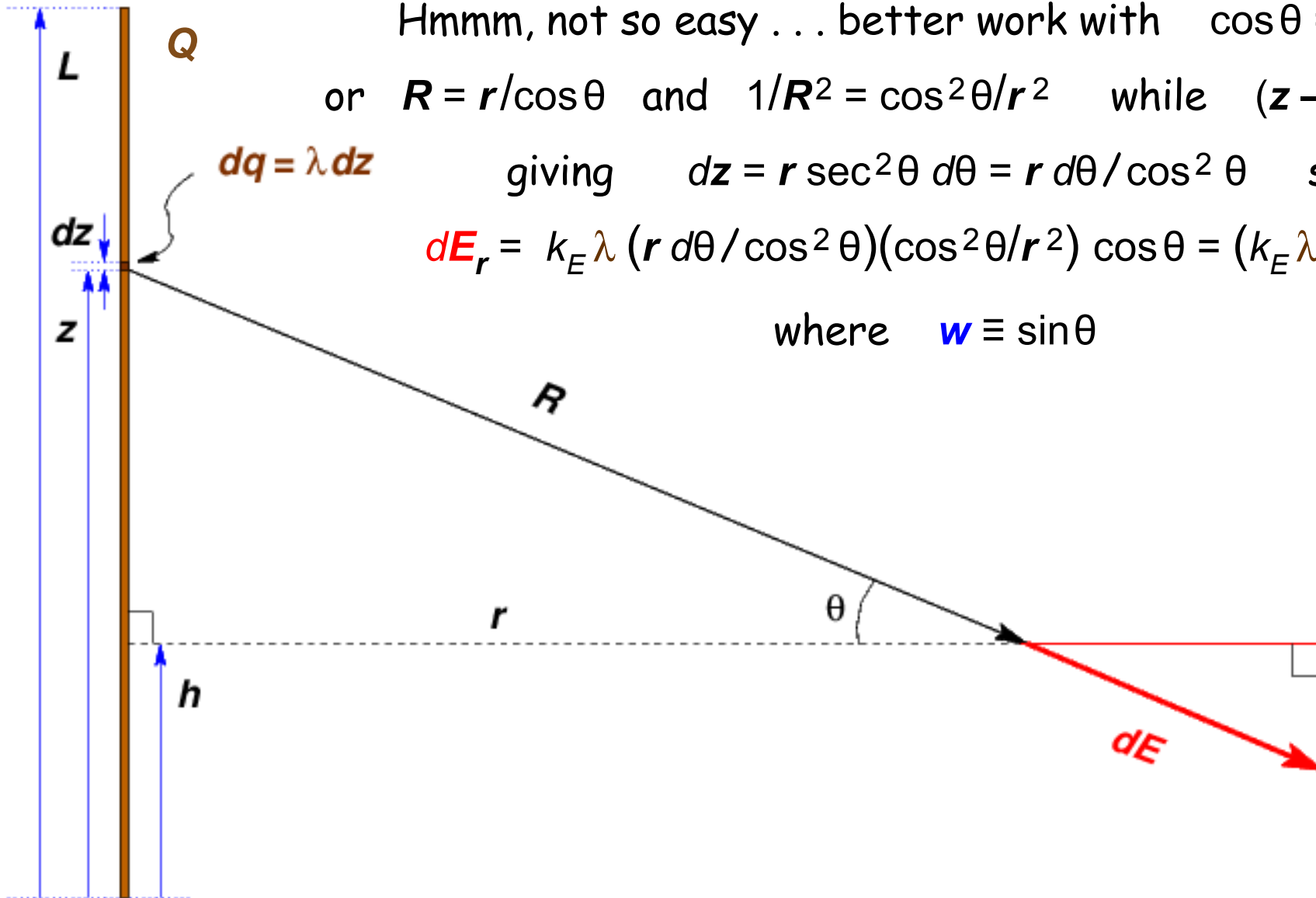
or $R = r/\cos \theta$ and $1/R^2 = \cos^2 \theta / r^2$ while $(z-h) = r \tan \theta$,

$$dq = \lambda dz$$

giving $dz = r \sec^2 \theta d\theta = r d\theta / \cos^2 \theta$ so that

$$dE_r = k_E \lambda (r d\theta / \cos^2 \theta) (\cos^2 \theta / r^2) \cos \theta = (k_E \lambda / r) dw,$$

where $w \equiv \sin \theta$



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$$\lambda = Q/L$$

$$d\mathbf{E}_r = k_E \lambda (r d\theta / \cos^2 \theta) (\cos^2 \theta / r^2) \cos \theta = (k_E \lambda / r) dw,$$

where $w \equiv \sin \theta$, giving

$$\mathbf{E}_r = (k_E \lambda / r) \int_{w(z=0)}^{w(z=L)} dw, \quad \text{or}$$

$$\mathbf{E}_r = (k_E \lambda / r) \left\{ (L - h) [r^2 + (L - h)^2]^{-1/2} + h [r^2 + h^2]^{-1/2} \right\}$$

