Maxwell's Equations

Gauss' Law for Electrostatics:

$$\vec{\nabla} \cdot \vec{D} = \rho \qquad \oiint_S \vec{D} \cdot d\vec{A} = Q_{\text{encl}}$$

Gauss' Law for Magnetostatics:

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \oiint_S \vec{B} \cdot d\vec{A} = Q_{\text{Magn}}$$

Faraday's Law:

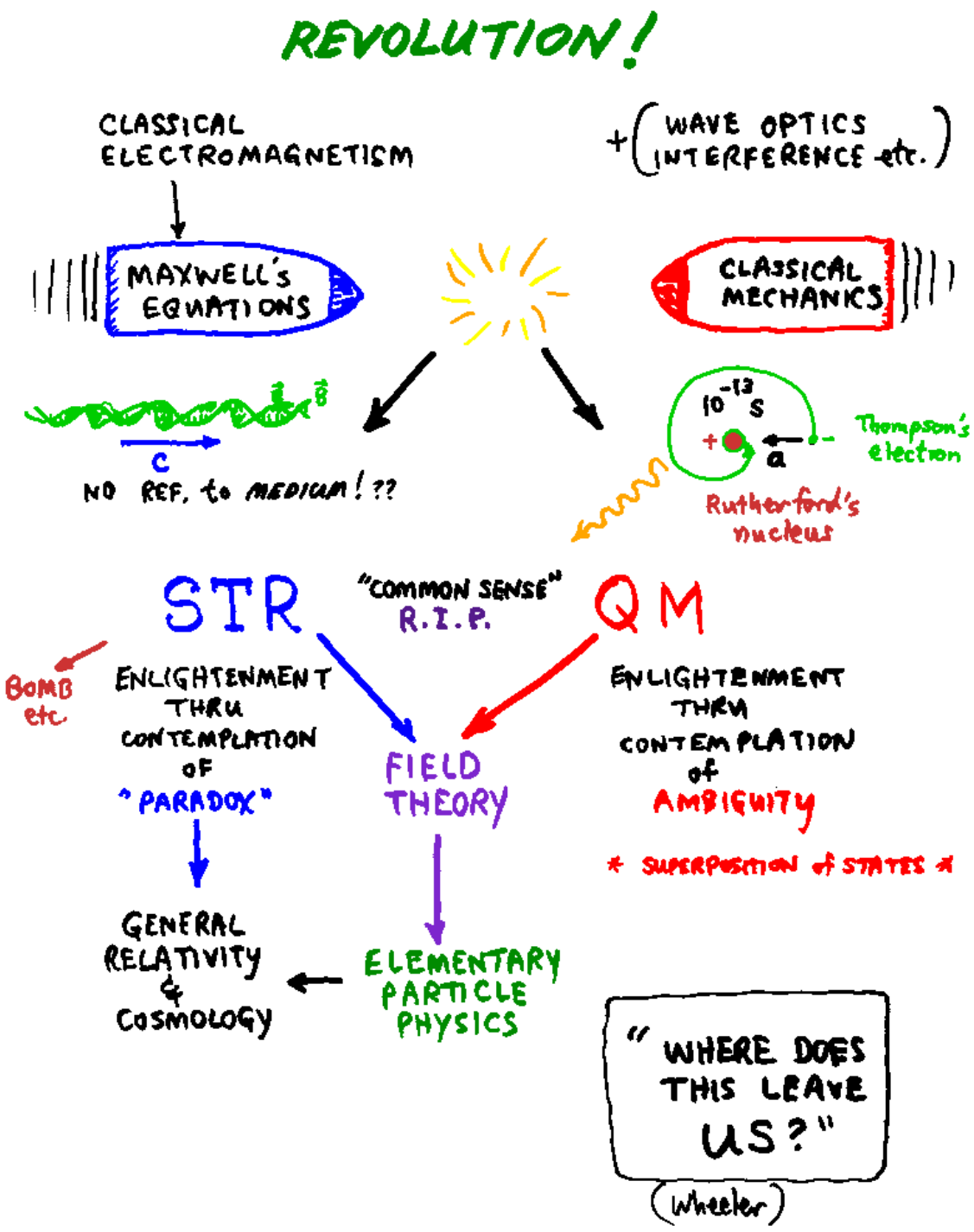
$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \qquad \oint_C \vec{E} \cdot d\vec{\ell} = -\frac{\partial}{\partial t} \iint_S \vec{B} \cdot d\vec{S}$$

Ampère's Law:

$$\vec{\nabla} \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J} \qquad \oint_C \vec{H} \cdot d\vec{\ell} = I + \frac{\partial}{\partial t} \iint_S \vec{D} \cdot d\vec{S}$$

Differential forms

Integral forms



Concept Map

At the beginning of the 20th Century, the conservative instincts of physicists were overcome by their devotion to honesty: the "laws" they had tested in the laboratory predicted phenomena in which common sense contradicted empirical fact. In such collisions, fact must win.

Without the guidance of common sense, the 20th Century was a wild ride!

In some ways, **Elementary Particle Physics** led the way into the unknown . . .

Matter

The fundamental constituents of matter are not a new topic. Around 2400 years ago, the Greek Democritos and his Atomist friends declared all matter to be composed of a small variety of indestructible units called **atoms**, while Heraclitus and the Epicureans insisted that all space was filled with a continuum of 4 **elements** (earth, air, fire and water) that combined and interacted in various ways to make all matter.

Who do you think was right?

QUANTUM MECHANICS causes havoc!

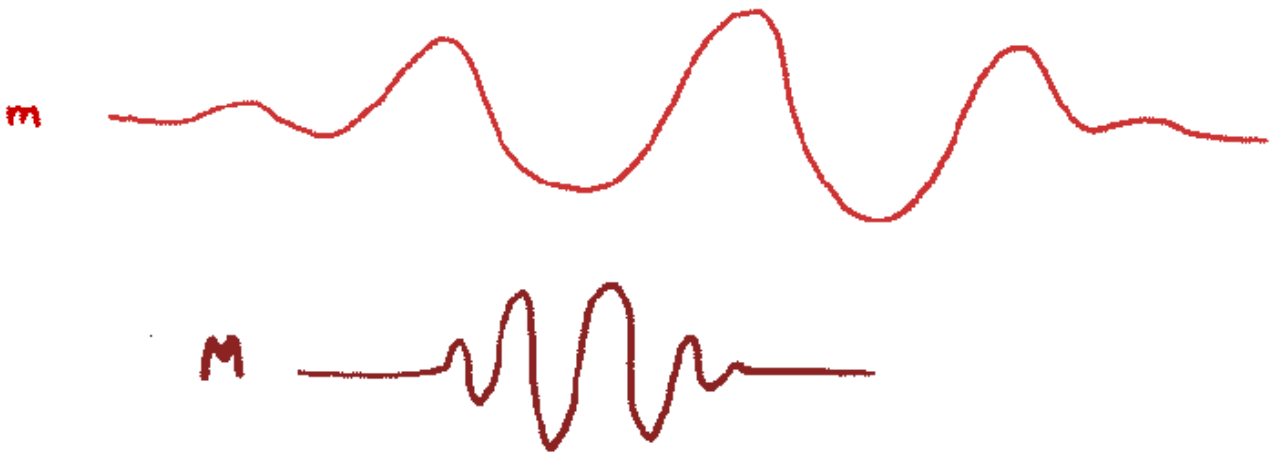
HEAVY PARTICLES are SMALLER (spatially) than LIGHT PARTICLES.

① All particles act like **WAVES** -- they have a SPATIAL **SIZE** CHARACTERIZED by their natural, intrinsic "**COMPTON WAVELENGTH**"

$$\lambda_c = \frac{\hbar}{mc}$$

... which is LONGER for SMALLER MASSES

⇒ LIGHTER particles are "**BIGGER**" than HEAVY ones.



② EXAMPLE: ELECTRON MICROSCOPES can resolve smaller objects than LIGHT microscopes because electrons are HEAVIER than photons.

Size Matters

Common sense dictates that **big**, **heavy** things are made up of smaller, lighter things. Unfortunately this is at odds with de Broglie's hypothesis and its extension to the intrinsic "size" of a particle-wave ("wavicle"?).

Experiments confirm this prediction.

Too bad for common sense.

Don't Fence Me In!

QUANTUM MECHANICS AND THE PROBLEM OF CONFINEMENT:

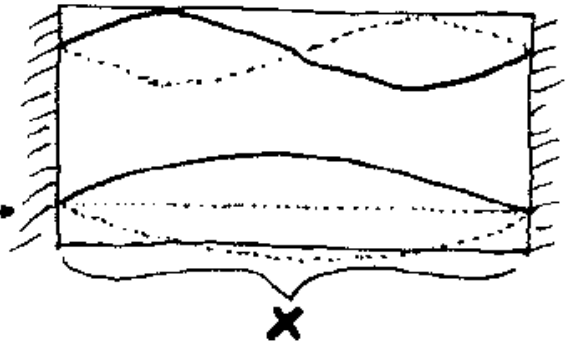
Another blow to common sense . . .

de Broglie: A PARTICLE IS A WAVE (and vice versa)

WAVELENGTH $\lambda = \frac{h}{p}$ (Planck's constant) / (MOMENTUM)

PARTICLE in a BOX:

"STANDING WAVES" ONLY!



LONGEST WAVELENGTH (SMALLEST MOMENTUM)

$\lambda_{MAX} \simeq 2X$

$\therefore p_{MIN} = \frac{h}{2X}$ → Heisenberg's UNCERTAINTY PRINCIPLE:

$\Delta x \Delta p \gtrsim \frac{h}{2}$

PROBLEM:

A PARTICLE COMPOSED OF "smaller" PARTICLES MUST CONFINE those "smaller" particles within its own Compton wavelength ⇒ A HEAVY PARTICLE IS A VERY SMALL "BOX" AND ITS COMPONENTS HAVE LARGE KINETIC ENERGIES!

(ESPECIALLY NUCLEI!)

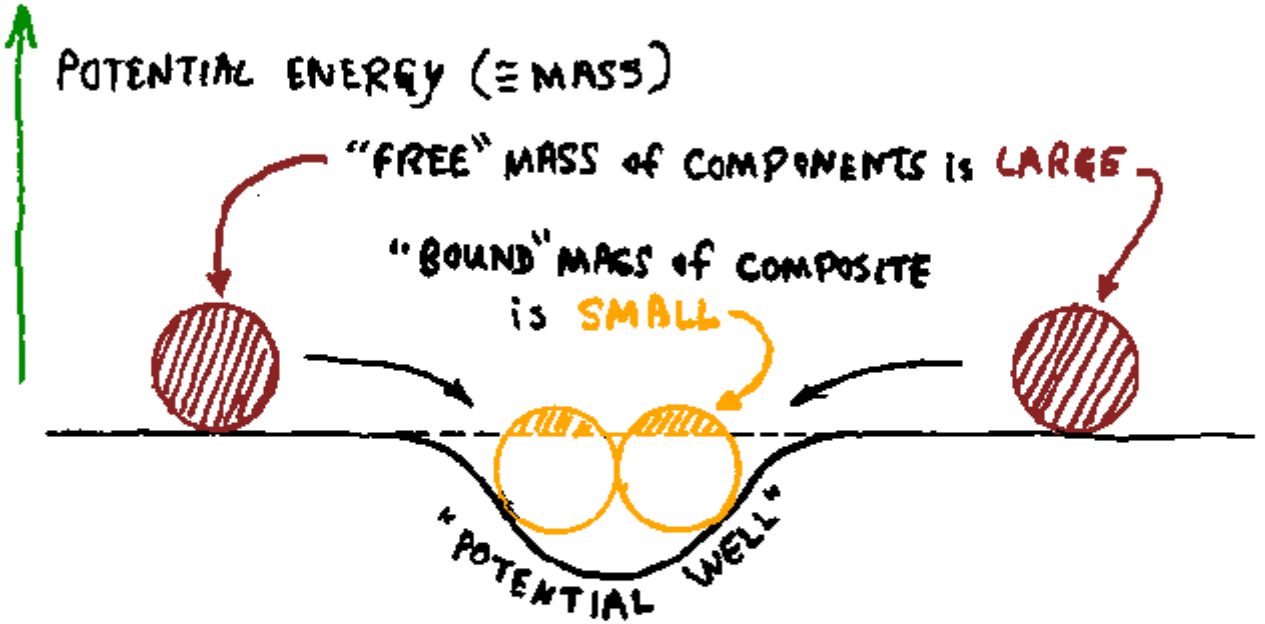
⇒ VERY STRONG FORCES MUST BE AT WORK!

Losing Mass

RELATIVITY causes havoc!

$E = mc^2$ INCLUDES POTENTIAL ENERGY.

. . . and another.



MASS can now "DISAPPEAR DOWN A POTENTIAL WELL"

⇒ LITTLE (*lightweight*) THINGS CAN BE MADE OUT OF BIG (*massive*) THINGS!

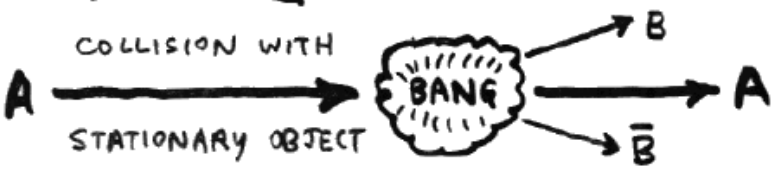
... all you need is a VERY STRONG POTENTIAL.

Field Theory

. . . and another.

RELATIVITY CAUSES MORE HAVOC:

$E = mc^2$ ⇒ PARTICLES can be CREATED & DESTROYED!



REVISED CONSERVATION LAW: $(E + mc^2) = \text{const.}$

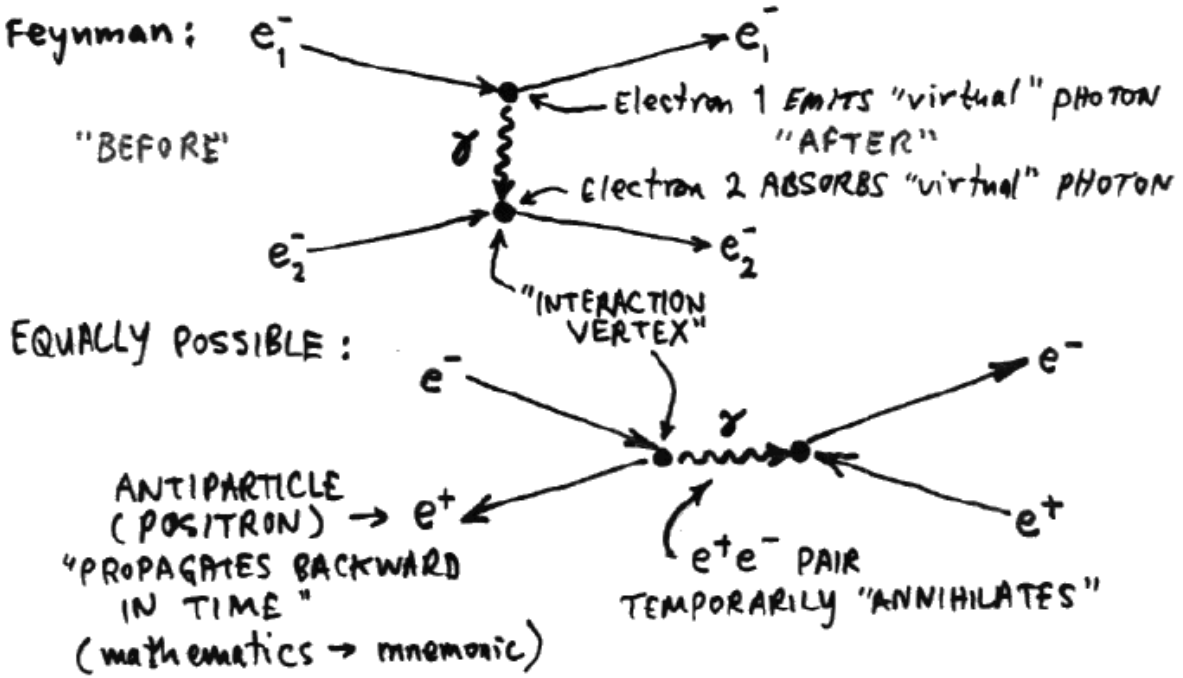
ALSO: "B NUMBER" = const.

⇒ "SECOND QUANTIZATION" (NUMBER of PARTICLES of each "FIELD" is a "QUANTUM NUMBER")

... RELATIVISTIC QUANTUM **FIELD THEORY!**

e.g., QUANTUM ELECTRODYNAMICS (QED)

as a model for how PARTICLES INTERACT:

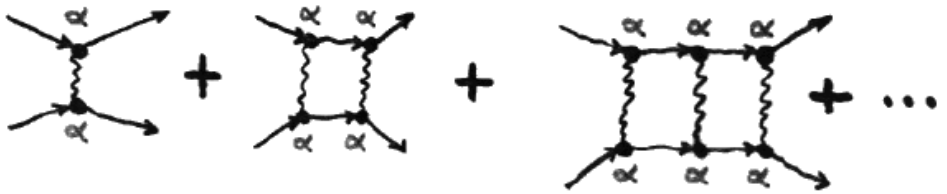


Perturbation Theory

. . . and another.

PROBLEMS WITH FIELD THEORY as a PERTURBATION THEORY:

QED:



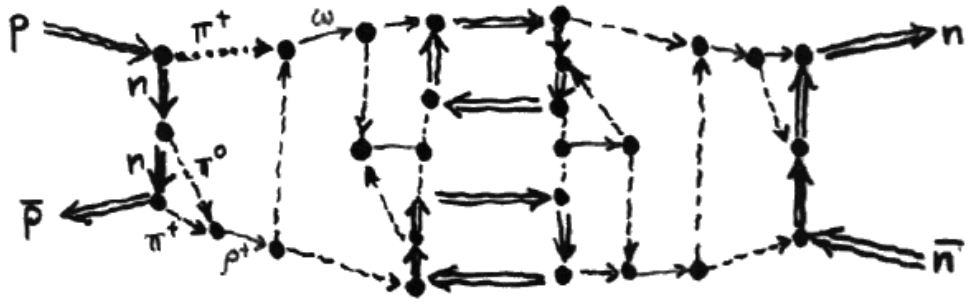
INTERACTION STRENGTH $\alpha = \frac{1}{137}$ at each VERTEX

⇒ MORE COMPLICATED DIAGRAMS are VERY IMPROBABLE ∴ CAN BE NEGLECTED for APPROXIMATIONS. (WHEW!)

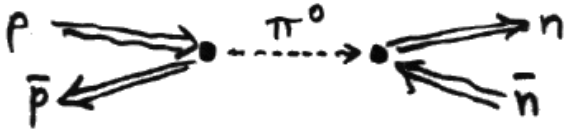
BUT

for STRONG INTERACTIONS, "STRENGTH" at each VERTEX ≈ 1.

THUS



IS INTRINSICALLY JUST AS IMPORTANT A PROCESS AS



-- "THIS CAUSES SOME CALCULATIONAL DIFFICULTIES."

Chew: "BOOTSTRAP THEORY" (EVERYTHING IS MADE UP of EVERYTHING ELSE.)

→Capra "Physics Mysticism"

ELEMENTARY PARTICLE TAXONOMY:

MOST EFFICIENT: SUCCESSIVE "ORTHOGONAL BINARY DISTINCTIONS":

1. STRONGLY INTERACTING? YES ← "HADRONS" (see below)

NO → "LEPTONS"

(e, ν_e)	(μ, ν_μ)	(τ, ν_τ)	... ??	"GENERATIONS"
0.511, 0	105.7, 0	1784, 0?	← MASS (MeV/c^2)	
∞, ∞ ?	$2.2\times10^{-6}, \infty$?	$5\times10^{-13}, \infty$?	← LIFETIME (sec)	

WEAK, ELECTROMAGNETIC (except ν's), GRAV. interactions only.
ALL "FERMIONS" (SPIN ½).

HADRONS:

2. FERMIONS (HALF-INTEGER SPIN) or BOSONS (INTEGER or ZERO SPIN)?

"BARYONS" ("HEAVY") → J= ½, 3/2, ... | J=0, 1, 2, ... ← "MESONS" ("MEDIUM-HEAVY")

ANGULAR MOMENTUM (SPIN) is CONSERVED
⇒ BARYONS cannot decay into MESONS alone!
∴ "BARYON NUMBER" is conserved.

STRONGLY INTERACTING ⇒ LIFETIMES VERY SHORT ($\sim 10^{-22}$ sec)
due to decay into LIGHTER hadrons. BUT SOME ARE MUCH STABLER than expected!
"STRANGE!"

⇒ A NEW, mysterious CONSERVED* QUANTITY called STRANGENESS, S

* But only by STRONG interactions.

Taxonomy

. . . and another.

Hadrons

. . . and another.

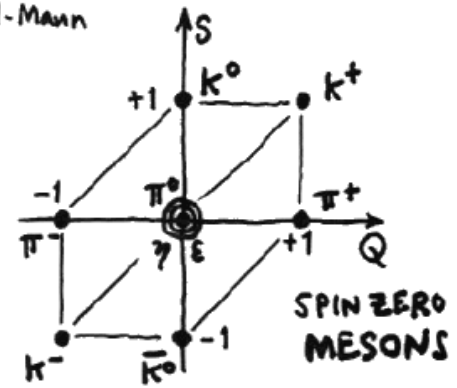
QUANTIFIABLE PROPERTIES of HADRONS:

M: MASS is the most obvious, but in the spirit of Democritos we would like to be able to EXPLAIN the mass in terms of OTHER properties and interactions.

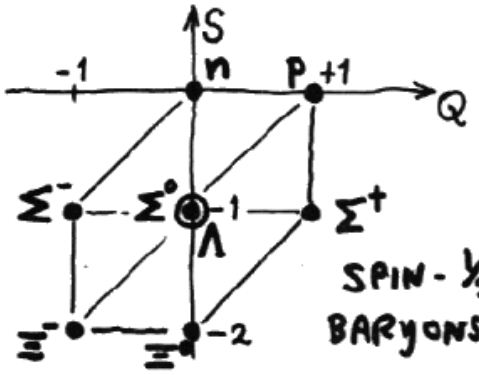
Q: CHARGE (electric) is easily measured in the lab and seems fairly "fundamental";

S: STRANGENESS is an interesting new property of particles that seems to be trying to tell us something. So...

Gell-Mann



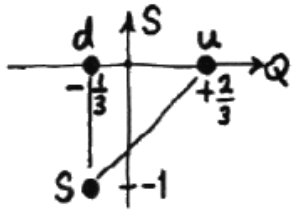
SPIN ZERO MESONS



SPIN - ½ BARYONS

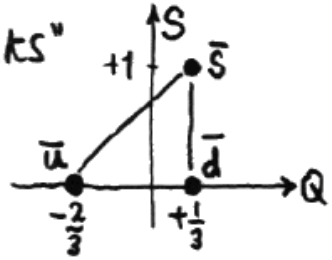
REPEATED PATTERNS $\stackrel{?}{\Rightarrow}$ SIGNIFICANCE? "EIGHTFOLD WAY" or SU(3)

"QUARKS"



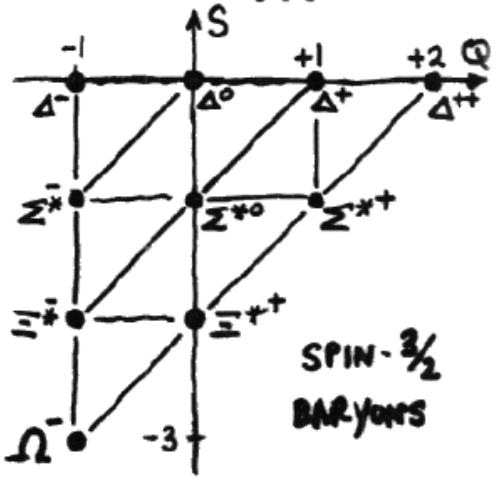
&

"ANTIQUARKS"



MESON = $q\bar{q}$

BARYON = qqq



SPIN - 3/2 BARYONS

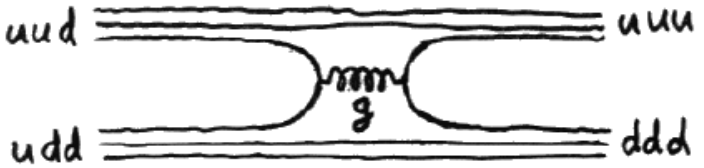
QUANTUM CHROMO DYNAMICS: PERTURBATION THEORY REVIVES!

Q: WHAT DO YOU DO WHEN YOUR THEORY PREDICTS INFINITE VALUES FOR PHYSICAL OBSERVABLES?

A: BUILD A MORE COMPLICATED THEORY IN WHICH ALL THE INFINITIES *AUTOMATICALLY CANCEL*!

CHEW's

becomes



gluons change quarks' "colors"

etc...

QUARK CONFINEMENT:

gluons can "BRANCH"

⇒ q-q POTENTIAL ENERGY *INCREASES* with DISTANCE r between quarks! (Contrast GRAV. or ELECT. potential $\sim \frac{1}{r}$)

∴ quarks bound LOOSELY at CLOSE RANGE, STRONGLY at LONG RANGE --- "BAG model" of NUCLEI + <u>NO FREE QUARKS</u>!

QCD

. . . and another.

JUST WHEN WE WERE STARTING TO GET COMPLACENT, ...

MORE QUARKS!

1974 Ting & Richter simultaneously discover an UNBELIEVABLY STABLE MESON with a mass of 3.1 GeV/c^2 (heavier than 3 protons together!) -- SHOULD BE ABLE TO DECAY "INSTANTLY" INTO MYRIAD LIGHTER PARTICLES!

"J/ψ"

⇒ some new conserved quantity (like "strangeness")

... call it "CHARM"; it belongs to CHARMED QUARKS (c)

$J/\psi = c\bar{c}$

-- MANY "EXCITED STATES" FOUND! "SPECTROSCOPY" of "CHARMONIUM":

"SUPERSTRONG" (GLUON) FORCE studied

Next found: "TOP" quark (t) via "TOPONIUM"

(MUCH HEAVIER STILL) (formerly "TRUTH")

Possible candidates at CERN (UA1) for "BOTTOM" quark (b) -- maybe next "BOTTOMONIUM"??

(formerly "BEAUTY")

WHERE WILL IT END?!

Charm, Truth and Beauty

. . . and another.

GENERATIONS & GUTs :

LEPTONS	QUARKS	FORCE CARRIERS	
e, ν_e	d, u	γ, g_{ud}	1st GENERATION
μ, ν_μ	s, c	$W^\pm, g_{cs}$	2nd GENERATION
τ, ν_τ	t, b	Z°, g_{tb}	3rd GENERATION
⋮ ??	⋮ ??	⋮ ??	⋮ ??

HEAVIER ↓

IS THIS REPEATED PATTERN "SIGNIFICANT"?

GRAND UNIFICATION THEORIES : "AT SUFFICIENTLY HIGH ENERGY, ALL INTERACTIONS (STRONG, ELECTROMAGNETIC, WEAK) BECOME EQUAL"

WHAT ABOUT GRAVITY?

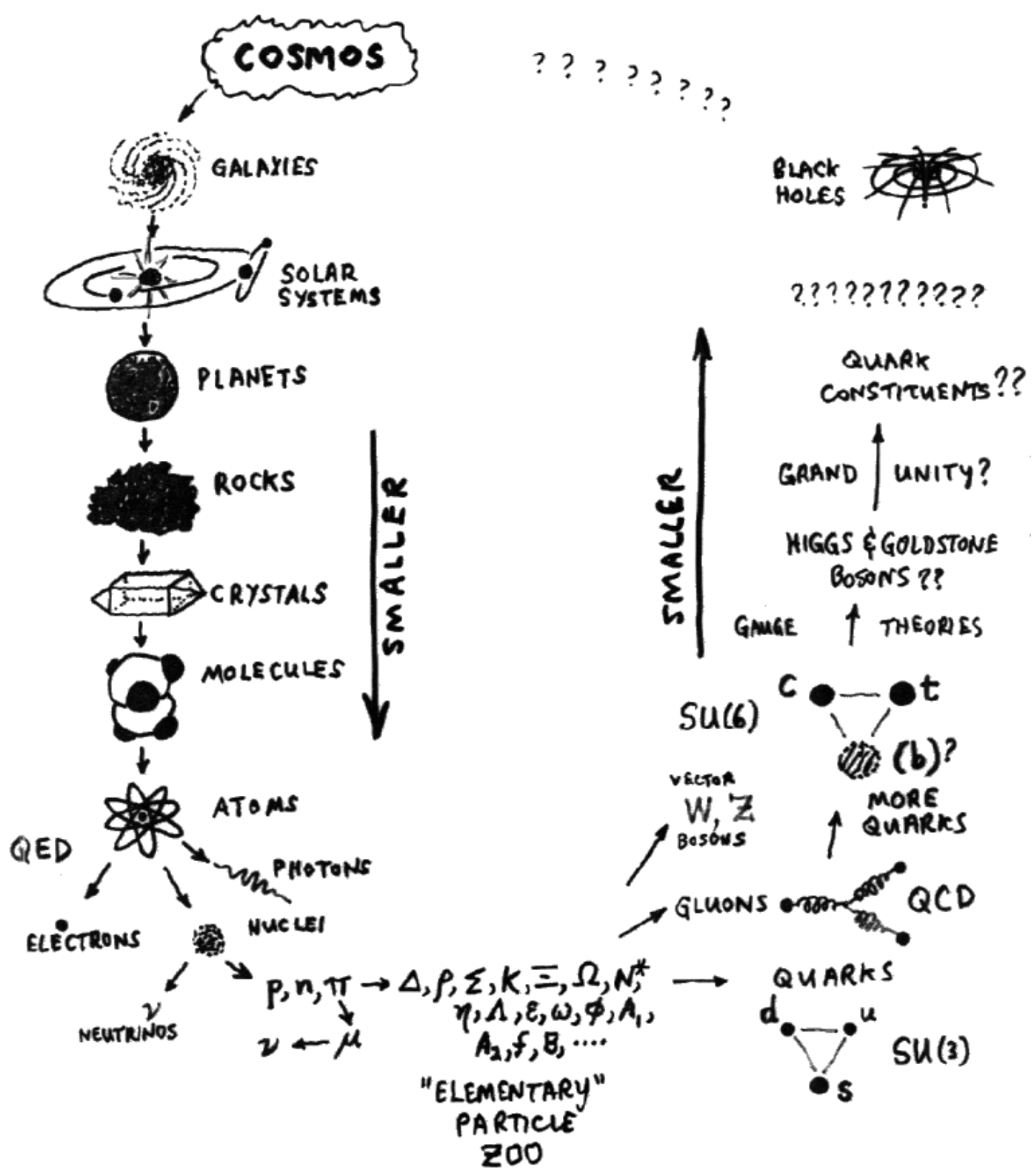
→ "SUPERSYMMETRY" THEORIES

(gravitinos, gravitons)

??

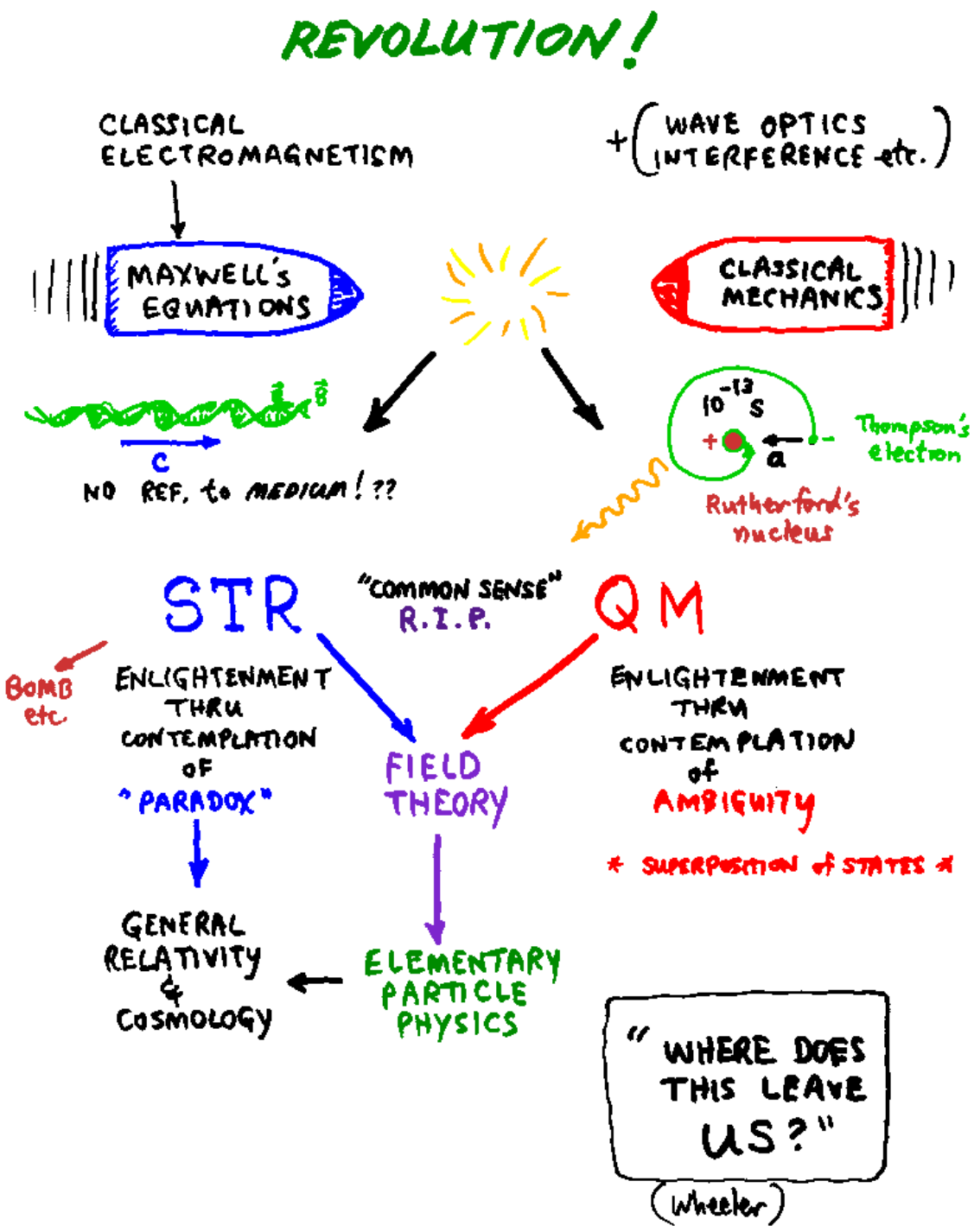
GUTs

. . . and another.


COSMOS
? ? ? ? ? ? ? ?
GALAXIES
SOLAR SYSTEMS
PLANETS
ROCKS
CRYSTALS
MOLECULES
ATOMS
SMALLER
QED
PHOTONS
ELECTRONS
NUCLEI
NEUTRINOS
p, n, π → Δ, ρ, Σ, K, Ξ, Ω, N*, η, Λ, ε, ω, φ, A1, A2, f, B, ...
ν ← μ
"ELEMENTARY" PARTICLE ZOO
BLACK HOLES
? ? ? ? ? ? ? ? ? ? ?
QUARK CONSTITUENTS ??
GRAND UNITY?
HIGGS & GOLDSTONE BOSONS ??
GAUGE THEORIES
SMALLER
SU(6)
c
t
(b)?
MORE QUARKS
VECTOR W, Z BOSONS
GLUONS
QCD
QUARKS
d
u
s
SU(3)

?

...



Concept Map

At the beginning of the 20th Century, the conservative instincts of physicists were overcome by their devotion to honesty: the "laws" they had tested in the laboratory predicted phenomena in which common sense contradicted empirical fact. In such collisions, fact must win.

Without the guidance of common sense, the 20th Century was a wild ride!

In some ways, **Elementary Particle Physics** led the way into the unknown . . .