BIOL/PHYS 438

Zoological Physics

- Logistics
- *Review* of ELECTROMAGNETISM
  - Phenomenology of I & $\boldsymbol{B}$:
    - Lorentz/Biot-Savart/Ampère/Faraday
  - *Magnets, Inductances & AC Circuits*
  - Maxwell's equations & Light

Logistics

Assignment 1: Solutions now online!

Assignment 2: Solutions now online!

Assignment 3: Solutions now online!

Assignment 4: Solutions online *soon!*

Assignment 5: Solutions now online!

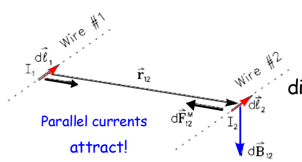
Assignment 6: Solutions online soon!

Next **Tue** & **Thu** in **Hebb 32**: ***Poster Sessions***

Direct Force Laws

The Coulomb force $\vec{F}^{E}_{12} = k_E \frac{q_1 q_2}{r^2_{12}} \hat{r}_{12}$ of charge q_1 on charge q_2 located at position $\boldsymbol{r}_{12}$ relative to the position of q_1 is already too complicated; instead we write $\boldsymbol{F}_{12} = q_2 \boldsymbol{E}$ and find $\boldsymbol{E}$ from **Coulomb's Law**:

$$\vec{E} = k_E \frac{Q}{r^2} \hat{r}$$



Even more intricate and confusing is the direct force between two *current elements:*

$$d\vec{F}^{M}_{12} = k_M \frac{I_1 I_2}{r^2_{12}} d\vec{\ell}_2 \times (d\vec{\ell}_1 \times \hat{r}_{12})$$

For this, we *presuppose* a magnetic field $\boldsymbol{B}$ at the position of $I_2 d\boldsymbol{\ell}_2 = q\, \boldsymbol{v}_2$ and combine both types of forces into one general electromagnetic force law:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

the **Lorentz Force**

The Lorentz Force

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

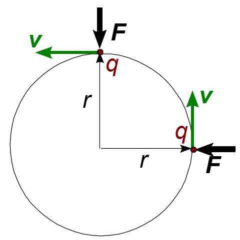
There are lots of applications of the Lorentz force, as you might expect. (After all, **force** is what we need to do some **work**!) We will look at:

- **Circulating Charges**: when $\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$ we get a force $\boldsymbol{F}$ that is perpendicular to both. This produces **uniform circular motion**.
  Cyclotrons: $p = qBr$ where p = momentum and r = orbit radius.
  Magnetic Mirrors: Magnetic forces **do no work**. Spiral paths reflect.
- **Velocity Selectors**: when $\boldsymbol{v}$ is perpendicular to both $\boldsymbol{E}$ and $\boldsymbol{B}$ we can adjust the ratio until $E/B = v$ so $F = 0$. If p is known, so is m.
- **Hall Effect**: charges moving down a conductor through a perpendicular magnetic field get swept sideways until a voltage builds up.
- **Rail Guns**: discharge a capacitor to make a huge current pulse....

The Cyclotron

When $\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$ we get a force $\boldsymbol{F}$ that is perpendicular to both. This is the familiar criterion for **uniform circular motion**. Recall

$mv^2/r = qvB$ or $p = qBr$ where $p = mv$.



Since $v = r\omega$ we have $m r \omega = qBr$

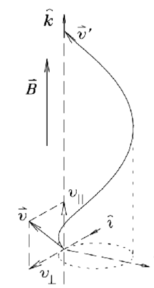
or

$\omega = qB/m$ = constant.

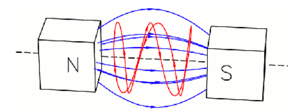
If the frequency of the charged particle orbit is constant, we can apply an accelerating voltage to the particles that reverses direction every half-orbit so that it is always in the right direction to make the particles go faster. This is what we call a **cyclotron**.

Magnetic Mirrors

Circulating Charges: when $\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$ we get a force $\boldsymbol{F}$ that is perpendicular to both. This produces *uniform circular motion*. This works on $v_\perp$, the *perpendicular component* of $\boldsymbol{v}$, in the same way. Any component $v_{||}$ of $\boldsymbol{v}$ that is *parallel* to $\boldsymbol{B}$ is *unaffected*. The net result is a *spiral* path:



What about *nonuniform* magnetic fields? If $v_{||}$ points into a region of stronger $\boldsymbol{B}$, then $v_{||}$ can't be parallel on both sides of the orbit, so $v_{||}$ gets smaller and eventually reverses direction. Remember, magnetic forces *do no work*, so this *reflection* is *perfectly elastic!* Many examples:

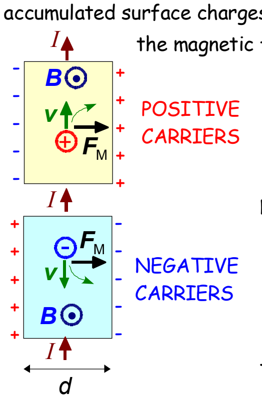


Aurorae
Tokamaks
Galactic accelerators

The Hall Effect: $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$

Charges moving down a conductor through a perpendicular magnetic field get swept sideways until an electric field E_{Hall} builds up due to the accumulated surface charges. When qE_{Hall} is just big enough to cancel the magnetic force $F_M = qvB$, the charges are no longer deflected. This implied a Hall field of $E_{Hall} = vB$, giving a **Hall voltage** of

$V_{Hall} = vBd$ across a conductor of width d.



Now, the current density is $J = nqv$, where n is the # of carriers per unitvolume, so $v = J/nq$ and thus $V_{Hall} = JBd/nq$

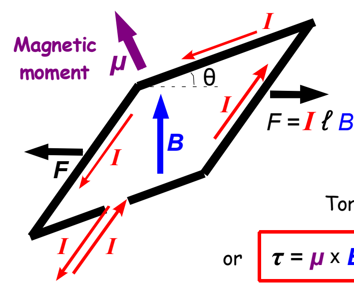
or $nq = JBd/V_{Hall}$.

This can be used to measure both q and n.

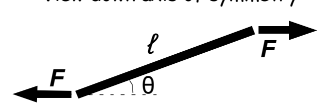
Torque on a Current Loop

$$d\boldsymbol{F} = I\, d\boldsymbol{\ell} \times \boldsymbol{B}$$

Picture a square loop ℓ on a side in a uniform magnetic field $\boldsymbol{B}$:



View down axis of symmetry:



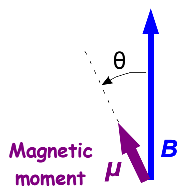
Torque $|\tau| = \ell F \sin\theta = I \ell^2 B \sin\theta$

or $\boldsymbol{\tau} = \boldsymbol{\mu} \times \boldsymbol{B}$ where $\boldsymbol{\mu} = I\boldsymbol{A}$ and $\boldsymbol{A} = \ell^2 \hat{\boldsymbol{n}}$ is the loop's **area** × a unit normal vector, **independent** of the loop's **shape**.

Energy of a Magnetic Moment

The torque $\boldsymbol{\tau} = \boldsymbol{\mu} \times \boldsymbol{B}$ "tries" to rotate $\boldsymbol{\mu}$ until it is parallel with $\boldsymbol{B}$.

As usual we calculate **angular work** as $dW = \tau\, d\theta$. If $d\theta$ is in the direction shown, dW is negative and the potential energy change $dU = -dW$ is positive. Since $|\tau| = \mu B \sin\theta$ is a function of θ, we must integrate $dU = \mu B \sin\theta\, d\theta$ or $dU = -\mu B\, du$ *where* $u \equiv \cos\theta$ to get $U = -\mu B \cos\theta$. This can be written

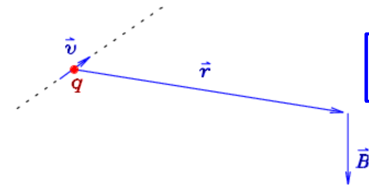


$$U = -\boldsymbol{\mu} \cdot \boldsymbol{B}$$

This expression should be familiar from Thermal Physics.
(Electrons, being negatively charged, have μ opposite to their spin.)

Law of Biot & Savart

So what is the *source* of the "magnetic field" $\boldsymbol{B}$? You might expect "magnetic charges" q_M analogous to our familiar electric charge q, but it turns out that (as far as anyone can tell, so far) *there aren't any!* Static magnetic fields *only* come from ***moving electric charges***. The rule is as follows: a charge q moving at velocity $\boldsymbol{v}$ produces a magnetic field $\boldsymbol{B}$ at position $\boldsymbol{r}$ (relative to q) given by

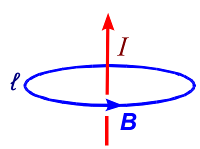


$$\boldsymbol{B} = k_M\, q\, (\boldsymbol{v} \times \boldsymbol{r}) / r^3$$

where $k_M = \mu_o/4\pi$
$= 10^{-7}$ T m/A

Ampère's Law

When we go to **calculate** the magnetic field $\boldsymbol{B}$ due to an electrical current, it is a hassle to use the Law of Biot & Savart. Usually we have a simple *symmetry* argument for why $\boldsymbol{B}$ must have the same magnitude B all the way around a path and be always parallel to that path; in which case the product of B times the length ℓ of the path is proportional to the net current I in ampères (A) "linking" the path:



$$B\,\ell = \mu_o\, I \qquad (\textit{Ampère's Law})$$

where we have introduced a new constant, μ_o, which is related to k_M.

Fundamental Constants

$c \equiv 2.99792458 \times 10^8$ m/s *exactly!*

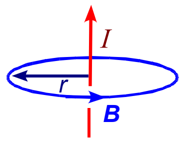
$k_E \equiv 1/4\pi\epsilon_0 = c^2 \times 10^{-7} \approx 8.98755 \times 10^9$ V·m·C^{-1}
$= 8.987551787368176 \times 10^9$ V·m·C^{-1} *exactly!*

$\epsilon_0 = 10^7 / 4\pi c^2 \approx 8.8542 \times 10^{-12}$ C^2·N^{-1}·m^{-2}

$k_M \equiv \mu_o/4\pi = 10^{-7}$ T·m·A^{-1} *exactly!*

$\mu_o = 4\pi \times 10^{-7} \approx 1.2566371 \times 10^{-6}$ T·m·A^{-1}

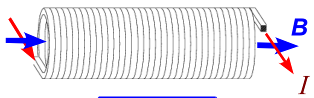
Magnetic Fields



Field at radius r due to a ***long straight wire*** carrying current I :

$$B = \mu I / 2\pi r$$

Axial field inside a long straight ***solenoid*** with n turns per unit length of wire carrying current I :



$$B = \mu n I$$

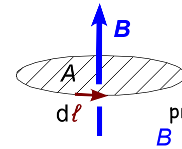
$\mu = (1 + \chi)\, \mu_o$, *where* χ is the magnetic susceptibility. In free space, $\chi = 0$ &

$\mu = \mu_o$.

This automatically takes care of the effect of magnetization in magnetic materials.

Flux & Inductance

$$\Phi \equiv A B$$



If we think of B as a sort of "flow" of "stuff", then we can define the magnetic flux Φ through an area A as the product of B and A, as long as B is essentially constant over A.

Whatever the shape of the current-carrying wires that make the field, B and Φ will always be proportional to the current I flowing through them. The constant of proportionality is called the ***inductance*** L of the magnet.

Definition of inductance L:

$$\Phi = L I$$

$I = \Phi / L$

$L = \Phi / I$

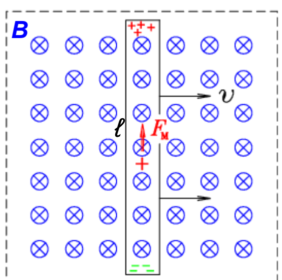
Note: L always has the form

$$L = (\text{const.}) \times (\mu_o / \text{length})$$

"Derivation" of Faraday's Law

We start with the Lorentz force on the charges in a conducting bar that moves through a uniform magnetic field $\boldsymbol{B}$ at a speed $\boldsymbol{v} \perp \boldsymbol{B}$.

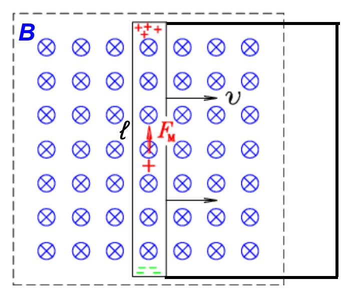
A + charge moving with the bar experiences an upward Lorentz force (recall the **Hall effect**) and will move to the top of the bar until enough + charges pile up at the top (& - charges at the bottom) to create an electric field $\boldsymbol{E}$ whose force cancels $\boldsymbol{F}_M$. For a bar of length ℓ, the voltage between the ends is $V = \ell \boldsymbol{E}$ or $V = \ell v B$. But ℓv is the **area** swept out by the bar per unit time, $\ell v = dA/dt$.



Time to recall magnetic flux!

Flux through a Loop

We have $V = \ell v B$ where ℓv is the **area** swept out by the bar per unit time, $\ell v = dA/dt$. If we define the magnetic flux $\Phi_M = \int \boldsymbol{B} \cdot d\boldsymbol{A}$ of the magnetic field passing through the closed loop formed by the bar and the rectangular path shown in black. This loop may be made of physical wires or it may only exist in our imagination; either way, the flux through it is changing at a rate given by $d\Phi_M/dt = \ell v B$. This is the same as the voltage across the bar. Since no other voltages act, it is also the voltage **around the loop**:



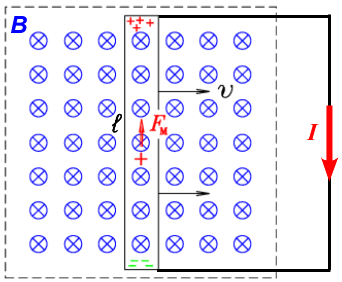
$$\mathcal{E}_{ind} = - d\Phi_M/dt$$

i.e. Faraday's Law !

(What does that - sign signify?)

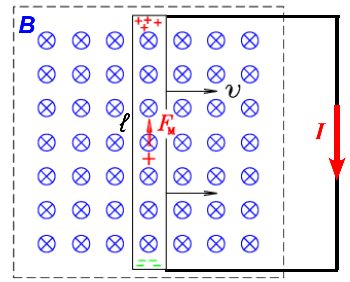
Lenz's Law

We have "derived" **Faraday's Law**, $\mathcal{E}_{ind} = -\, d\Phi_M/dt$. What does the - sign signify? This has to do with the **direction** of the induced EMF $\mathcal{E}_{ind}$: If we imagine for the moment that the black path actually is a **wire**, then a **current** can flow around the loop due to $\mathcal{E}_{ind}$. (We imagine the wire to have a small **resistance**, to avoid confusing aspects of superconductivity.) The current will flow **clockwise** here, so as to reunite the **+** & **-** charges. This current makes its own magnetic field into the page, and thus **adds** to the net flux through the loop in that direction, which **was decreasing** as the loop was pulled out of the field. So <u>the induced EMF "tries" to make a current flow to **counteract the change in flux**</u>. (**Lenz's Law**)



Magic!

We have "derived" **Faraday's Law**, $\mathcal{E}_{ind} = -\, d\Phi_M/dt$, and **Lenz's Law** for the particular scenario shown below. This may seem an artificial way of expressing what is obvious from the simple Lorentz force law. What's so amazing about a simple change of terminology? The "magic" of this Law is that it applies (and works!) equally in situations that bear no resemblance to this example. If there is **no physical motion** at all, but only a change in the **strength** of the magnetic field, we still get an induced EMF according to the same equation. This accounts for the enormous impact of "electric power" on the modern world. It also leads to our understanding of the nature of light itself.



Inductance as "Magnetic Inertia"

We call $\Delta V = -\, L\, dI/dt$ the "ElectroMotive Force" (**EMF**) across an inductance. If you actually think of ΔV as a sort of pseudo**force**, then it is easy to think of I as a sort of **velocity** and L as the analogue of **mass**, providing "inertia" to the circuit. This metaphor offers an excellent way to understand what happens with inductances in circuits.

"Adding" Inductances

<u>In PARALLEL</u>:

Charge is conserved

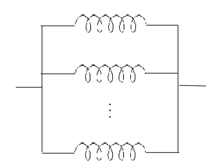
$\Rightarrow I_{tot} = \Sigma_i I_i = \Sigma_i \Phi_i / L_i$

Same ΔV across each L_i

$\Delta V = -\, L_i\, dI_i/dt$

$= -\, L_{eff}\, dI_{tot}/dt$

or $L_{eff} = 1/\Sigma_i L_i^{-1}$ -- *i.e.* **ADD INVERSES!**



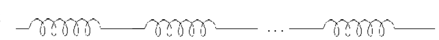
Definition of **inductance** L :

$\Phi = L\, I$

$I = \Phi / L$

$L = \Phi / I$

<u>In SERIES</u>:



The same current goes through all the coils: $I = I_i$. Thus the "Voltage drops" $\Delta V_i = -\, L_i\, dI/dt$ add up: $\Delta V_{tot} = -\, \Sigma_i L_i\, dI/dt$

or $L_{eff} = \Sigma_i L_i$ -- *i.e.* **ADD INDUCTANCES!**

Magnetic Energy Storage

It takes electrical work $dW = V\,dQ$ to "push" a bit of charge dQ through an inductance L against the opposing EMF $V = -L\,dI/dt$. This work is "stored" in the inductance as

$$dU_M = -\,dW = -L\,(dI/dt)\,dQ = -L\,(dQ/dt)\,dI = -L\,I\,dI\,.$$

If we start with an inductance with no current through it and add up the energy stored at each increase of dI [*i.e.* **integrate**], we get

$$U_M = ½\,L\,I^2$$

just like with an accelerated mass -- L is like the "magnetic mass".

Combined with the formulae for the *volume* of the coil and the magnetic field B as a function of L and I, this gives

$$U_M/\mathrm{Vol} \equiv u_M = ½\,B^2/\mu_o$$

Maxwell's Equations

Gauss' Law for Electrostatics:

$$\vec{\nabla}\cdot\vec{D} = \rho \qquad \oiint_S \vec{D}\cdot d\vec{A} = Q_{\mathrm{encl}}$$

Gauss' Law for Magnetostatics:

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \oiint_S \vec{B}\cdot d\vec{A} = Q_{\mathrm{Magn}}$$

Faraday's Law:

$$\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0 \qquad \oint_C \vec{E}\cdot d\vec{\ell} = -\frac{\partial}{\partial t}\iint_S \vec{B}\cdot d\vec{S}$$

Ampère's Law:

$$\vec{\nabla}\times\vec{H} - \frac{\partial\vec{D}}{\partial t} = \vec{J} \qquad \oint_C \vec{H}\cdot d\vec{\ell} = I + \frac{\partial}{\partial t}\iint_S \vec{D}\cdot d\vec{S}$$

Differential forms | Integral forms

The Electromagnetic Spectrum